The depth required for containment of a fixed fraction of the energy also increases logarithmically with incident particle energy. The thickness of iron required for 95% (99%) containment of cascades induced by single hadrons is shown in Fig. 12.1 [35]. Two of the sets of data are from large neutrino experiments, while the third is from a commonly used parametrization. Depths as measured in nuclear interaction lengths presumably scale to other materials. From the same data it can be concluded that the requirement that 95% of the energy in 95% of the showers be contained requires 40 to 50 cm (2.4 to 3.0 λ_I) more material material than for an average 95% containment.

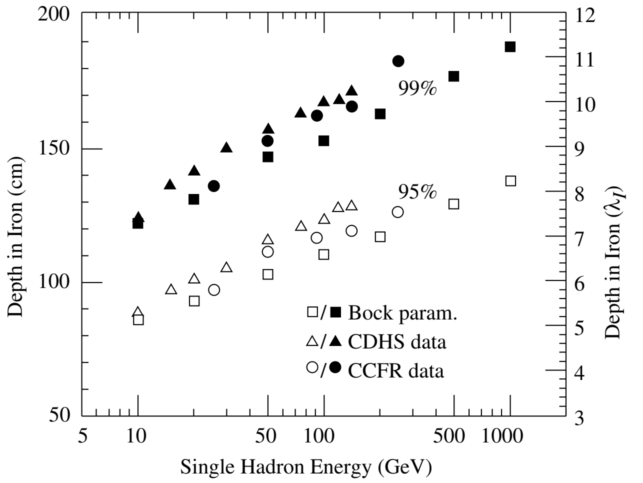

Figure 12.1: Required calorimeter thickness for 95% and 99% hadronic cascade containment in iron, on the basis of data from two large neutrino detectors and the parametrization of Bock *et al.* [35].

The transverse dimensions of hadronic showers also scale as λ_I, although most of the energy is contained in a narrow core.

The energy deposit in a hadronic cascade consists of a prompt EM component due to π^0 production and a slower component mainly due to low-energy hadronic activity. In general, these energy depositions are converted to electrical signals with different efficiencies [36]. The ratio of the conversion efficiencies is usually called the intrinsic e/h ratio. If $e/h = 1.0$ the calorimeter is said to be *compensating.* If it differs from unity by more than 5% or 10%, detector performance is compromised because of fluctuations in the π^0 content of the cascades. Problems include:

a) A skewed signal distribution;

b) A response ratio for electrons and hadrons (the "e/π ratio") which is different from unity and depends upon energy;

c) A nonlinear response to hadrons (the response per GeV is proportional to the reciprocal of e/π);

d) A constant contribution to detector resolution, almost proportional to the degree of noncompensation. The coefficient relating the constant term to $|1 - e/h|$ is 14% according to FLUKA simulations, and 21% according to Wigman's calculations [33].

In most cases e/h is greater than unity, particularly if little hydrogen is present or if the gate time is short. This is because much of the low-energy hadronic energy is "hidden" in nuclear binding energy release, low-energy spallation products, *etc.* Partial correction for these losses occurs in a sampling calorimeter with thick plates, because a disproportionate fraction of electromagnetic energy is deposited in the inactive region. For this reason, it is very unlikely that a fully sensitive detector such as BGO or glass can be made compensating.

Compensation has been demonstrated in calorimeters with 2.5 mm scintillator sheets sandwiched between 3 mm depleted uranium plates [38] or 10 mm lead plates [39]; resolutions σ/E of $0.34/\sqrt{E}$ and $0.44/\sqrt{E}$ were obtained for these cases (E in GeV). The former was shown to be linear to within 2% over three orders of magnitude in energy, with approximately Gaussian signal distributions.

dE/dx resolution in argon. Particle identification by dE/dx is dependent on the width of the distribution. For relativistic incident particles with charge e in a multiple-sample Ar gas counter with no lead [40],

$$\left.\frac{dE}{dx}\right|_{\rm FWHM} \Big/ \left.\frac{dE}{dx}\right|_{\rm most\ probable} = 0.96\, N^{-0.46}\,(xp)^{-0.32} \ , \qquad (12.15)$$

where N = number of samples, x = thickness per sample (cm), p = pressure (atm.). Most commonly used chamber gases (except Xe) give approximately the same resolution.

Free electron drift velocities in liquid ionization chambers [41–44]. Velocity as a function of electric field strength is given in Fig. 12.2.

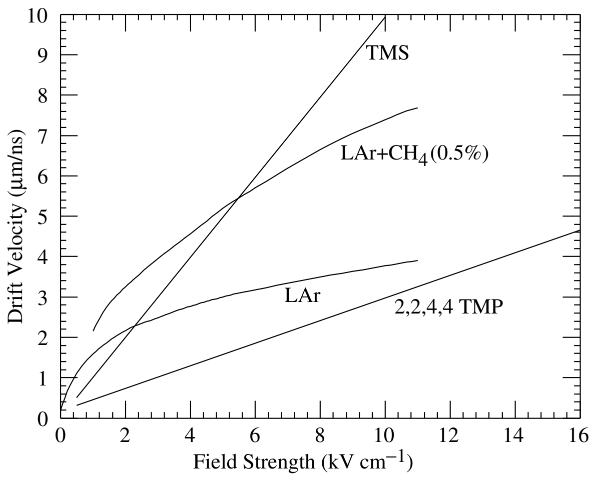

Figure 12.2: Electron drift velocity as a function of field strength for commonly used liquids.

12.8. Measurement of particle momenta in a uniform magnetic field [45]

The trajectory of a particle with momentum p (in GeV/c) and charge ze in a constant magnetic field $\vec{B}$ is a helix, with radius of curvature R and pitch angle λ. The radius of curvature and momentum component perpendicular to $\vec{B}$ are related by

$$p\cos\lambda = 0.3\, z\, B\, R \ , \qquad (12.16)$$

where B is in tesla and R is in meters.

The distribution of measurements of the curvature $k \equiv 1/R$ is approximately Gaussian. The curvature error for a large number of uniformly spaced measurements on the trajectory of a charged particle in a uniform magnetic field can be approximated by

$$(\delta k)^2 = (\delta k_{\rm res})^2 + (\delta k_{\rm ms})^2 \ , \qquad (12.17)$$

where δk = curvature error

$\delta k_{\rm res}$ = curvature error due to finite measurement resolution

$\delta k_{\rm ms}$ = curvature error due to multiple scattering.

If many (≥ 10) uniformly spaced position measurements are made along a trajectory in a uniform medium,

$$\delta k_{\rm res} = \frac{\epsilon}{L'^2}\sqrt{\frac{720}{N+4}} \ , \qquad (12.18)$$

where N = number of points measured along track