L' = the projected length of the track onto the bending plane
ϵ = measurement error for each point, perpendicular to the trajectory.

If a vertex constraint is applied at the origin of the track, the coefficient under the radical becomes 320.

The contribution due to multiple Coulomb scattering is approximately

$$\delta k_{\rm ms} \approx \frac{(0.016)({\rm GeV}/c)z}{Lp\beta\cos^2\lambda}\sqrt{\frac{L}{X_0}}\ , \tag{12.19}$$

where p = momentum (GeV/c)
z = charge of incident particle in units of e
L = the total track length
X_0 = radiation length of the scattering medium (in units of length; the X_0 defined elsewhere must be multiplied by density)
β = the kinematic variable v/c.

More accurate approximations for multiple scattering may be found in the section on Passage of Particles Through Matter (Sec. 10 of this *Review*). The contribution to the curvature error is given approximately by $\delta k_{\rm ms} \approx 8s^{\rm rms}_{\rm plane}/L^2$, where $s^{\rm rms}_{\rm plane}$ is defined there.

12.9. Superconducting solenoids for collider detectors

12.9.1. ***Basic (approximate) equations***: In all cases SI units are assumed, so that B is in tesla, E is in joules, dimensions are in meters, and $\mu_0 = 4\pi \times 10^{-7}$.

Magnetic field. The magnetic field at the center of a solenoid of length L and radius R, having N total turns and a current I is

$$B(0,0) = \frac{\mu_0 NI}{\sqrt{L^2+4R^2}}\ . \tag{12.20}$$

Stored energy. The energy stored in the magnetic field of any magnet is calculated by integrating B^2 over all space:

$$E = \frac{1}{2\mu_0}\int B^2 dV\ . \tag{12.21}$$

For a solenoid with an iron flux return in which the magnetic field is $< 2T$, the field in the aperture is approximately uniform and equal to $\mu_0 NI/L$. If the thickness of the coil is small, (which is the case if it is superconducting), then

$$E \approx (\pi/2\mu_0)B^2R^2L\ . \tag{12.22}$$

Cost of a superconducting solenoid [46]:

$$\text{Cost (in M\$)} = 0.523\,[(E/(1\ \text{MJ})]^{0.662} \tag{12.23}$$

Magnetostatic computer programs. It is too difficult to solve the Biot-Savart equation for a magnetic circuit which includes iron components and so iterative computer programs are used. These include POISSON, TOSCA [47], and ANSYS [48].

12.9.2. ***Scaling laws for thin solenoids***: For a detector in which the calorimetry is outside the aperture of the solenoid, the coil must be thin in terms of radiation and absorption lengths. This usually means that the coil is superconducting and that the vacuum vessel encasing it is of minimum real thickness and fabricated of a material with long radiation length. There are two major contributers to the thickness of a thin solenoid:

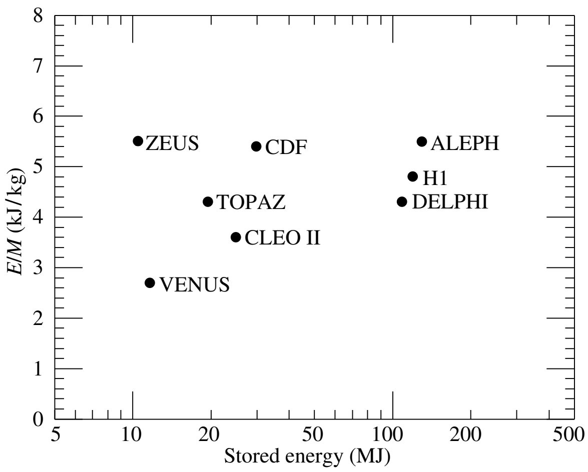

Figure 12.3: Ratio of stored energy to cold mass for existing thin detector solenoids.

1. The conductor, consisting of the current-carrying superconducting material (usually Cu/Nb-Ti) and the quench protecting stabilizer (usually aluminum), is wound on the inside of a structural support cylinder (usually aluminum also). This package typically represents about 60% of the total thickness in radiation lengths. The thickness scales approximately as B^2R.
2. Approximately another 25% of the thickness of the magnet comes from the outer cylindrical shell of the vacuum vessel. Since this shell is susceptible to buckling collapse, its thickness is determined by the diameter, length, and the modulus of the material of which it is fabricated. When designing this shell to a typical standard, the real thickness is

$$t = P_c D^{2.5}[(L/D) - 0.45(t/D)^{0.5}]/2.6Y^{0.4}\ , \tag{12.24}$$

where t = shell thickness (in), D = shell diameter (in), L = shell length (in), Y = modulus of elasticity (psi), and P_c = design collapse pressure (= 30 psi). For most large-diameter detector solenoids, the thickness to within a few percent is given by [49]

$$t = P_c D^{2.5}(L/D)/2.6Y^{0.4}\ . \tag{12.25}$$

12.9.3. ***Properties of collider detector solenoids***: The physical dimensions, central field, stored energy and thickness in radiation lengths normal to the beam line of the superconducting solenoids associated with the major colliders are given in Table 12.5.

Table 12.5: Properties of superconducting collider detector solenoids.

Experiment–Lab	Field (T)	Bore Dia (m)	Length (m)	Energy (MJ)	Thickness (X_0)
CDF-Fermilab	1.5	2.86	5.07	30	0.86
Topaz–KEK	1.2	2.72	5.4	19.5	0.70
Venus–KEK	0.75	3.4	5.64	12	0.52
Cleo II–Cornell	1.5	2.9	3.8	25	2.5
Aleph–CERN	1.5	5.0	7.0	130	1.7
Delphi–CERN	1.2	5.2	7.4	109	4.0
H1–DESY	1.2	5.2	5.75	120	1.2
Zeus–DESY	1.8	1.72	2.85	10.5	0.9

The ratio of stored energy to cold mass (E/M) is a useful performance measure. One would like the cold mass to be as small as possible to minimize the thickness, but temperature rise during a quench must also be minimized. Ratios as large as 8 kJ/kg may be possible (final temperature of 80 K after a fast quench with homogenous energy dump), but some contingency is desirable. This quantity is shown as a function of total stored energy for some major collider detectors in Fig. 12.3.