19.1. Impedances (SI units)

ρ = resistivity at room temperature in $10^{-8}\ \Omega$ m:

$\sim$ 1.7 for Cu	$\sim$ 5.5 for W
$\sim$ 2.4 for Au	$\sim$ 73 for SS 304
$\sim$ 2.8 for Al	$\sim$ 100 for Nichrome

(Al alloys may have double the Al value.)

For alternating currents, instantaneous current I, voltage V, angular frequency ω:

$$V = V_0\, e^{j\omega t} = ZI\ . \tag{19.1}$$

Impedance of self-inductance L: $Z = j\omega L$.

Impedance of capacitance C: $Z = 1/j\omega C$.

Impedance of free space: $Z = \sqrt{\mu_0/\epsilon_0} = 376.7\ \Omega$.

Impedance per unit length of a flat conductor of width w (high frequency, ν):

$$Z = \frac{(1+j)\,\rho}{w\,\delta}\ , \quad \text{where } \delta = \text{effective skin depth}\ ; \tag{19.2}$$

$$\delta = \sqrt{\frac{\rho}{\pi\nu\mu}} \approx \frac{6.6\ \text{cm}}{\sqrt{\nu\,(\text{Hz})}} \quad \text{for Cu}\ . \tag{19.3}$$

19.2. Capacitance $\widehat{C}$ and inductance $\widehat{L}$ per unit length (SI units)

Flat rectangular plates of width w, separated by $d \ll w$:

$$\widehat{C} = \epsilon\,\frac{w}{d}\ ; \qquad \widehat{L} = \mu\,\frac{d}{w}\ ; \tag{19.4}$$

$$\frac{\epsilon}{\epsilon_0} = 2 \text{ to } 6 \text{ for plastics; } 4 \text{ to } 8 \text{ for porcelain, glasses.} \tag{19.5}$$

Coaxial cable of inner radius r_1, outer radius r_2:

$$\widehat{C} = \frac{2\pi\,\epsilon}{\ln\,(r_2/r_1)}\ ; \qquad \widehat{L} = \frac{\mu}{2\pi}\,\ln\,(r_2/r_1)\ . \tag{19.6}$$

Transmission lines (no loss):

$$\text{Impedance: } Z = \sqrt{\widehat{L}/\widehat{C}}\ . \tag{19.7}$$

$$\text{Velocity: } v = 1/\sqrt{\widehat{L}\,\widehat{C}} = 1/\sqrt{\mu\,\epsilon}\ . \tag{19.8}$$

19.3. Synchrotron radiation (CGS units)

For a particle of charge e, velocity $v = \beta c$, and energy $E = \gamma mc^2$, traveling in a circular orbit of radius R, the energy loss per revolution δE is

$$\delta E = \frac{4\pi}{3}\,\frac{e^2}{R}\,\beta^3\,\gamma^4\ . \tag{19.9}$$

For high-energy electrons or positrons ($\beta \approx 1$), this becomes

$$\delta E\ (\text{in MeV}) \approx 0.0885\ [E(\text{in GeV})]^4/R(\text{in m})\ . \tag{19.10}$$

For $\gamma \gg 1$, the energy radiated per revolution into the photon energy interval $d(\hbar\omega)$ is

$$dI = \frac{8\pi}{9}\alpha\,\gamma\,F(\omega/\omega_c)\,d(\hbar\omega)\ , \tag{19.11}$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant and

$$\omega_c = \frac{3\gamma^3 c}{2R} \tag{19.12}$$

is the critical frequency. The normalized function $F(y)$ is

$$F(y) = \frac{9}{8\pi}\sqrt{3}\,y\int_y^\infty K_{5/3}\,(x)\ dx\ , \tag{19.13}$$

where $K_{5/3}\,(x)$ is a modified Bessel function of the third kind. For electrons or positrons,

$$\hbar\omega_c\,(\text{in keV}) \approx 2.22\ [E(\text{in GeV})]^3/R(\text{in m})\ . \tag{19.14}$$

Fig. 19.1 shows $F(y)$ over the important range of y.

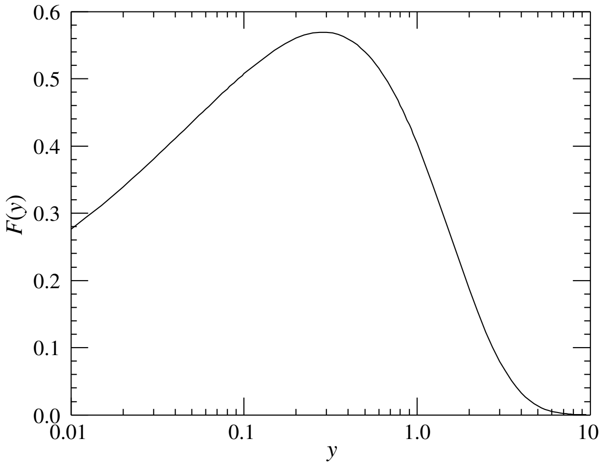

Figure 19.1: The normalized synchrotron radiation spectrum $F(y)$.

For $\gamma \gg 1$ and $\omega \ll \omega_c$,

$$\frac{dI}{d(\hbar\omega)} \approx 3.3\alpha\,(\omega R/c)^{1/3}\ , \tag{19.15}$$

whereas for

$\gamma \gg 1$ and $\omega \gtrsim 3\omega_c$,

$$\frac{dI}{d(\hbar\omega)} \approx \sqrt{\frac{3\pi}{2}}\,\alpha\,\gamma\left(\frac{\omega}{\omega_c}\right)^{1/2} e^{-\omega/\omega_c}\left[1 + \frac{55}{72}\frac{\omega_c}{\omega} + \ldots\right]\ . \tag{19.16}$$

The radiation is confined to angles $\lesssim 1/\gamma$ relative to the instantaneous direction of motion.

See J.D. Jackson, *Classical Electrodynamics*, 2[nd] edition (John Wiley & Sons, New York, 1975) for more formulae and details. In his book, Jackson uses a definition of ω_c that is twice as large as the customary one given above.