23.5.3. ***Partial waves***: The amplitude in the center of mass for elastic scattering of spinless particles may be expanded in Legendre polynomials

$$f(k,\theta) = \frac{1}{k}\sum_{\ell}(2\ell+1)a_\ell P_\ell(\cos\theta) \ , \tag{23.49}$$

where k is the c.m. momentum, θ is the c.m. scattering angle, $a_\ell = (\eta_\ell e^{2i\delta_\ell} - 1)/2i$, $0 \le \eta_\ell \le 1$, and δ_ℓ is the phase shift of the ℓ^{th} partial wave. For purely elastic scattering, $\eta_\ell = 1$. The differential cross section is

$$\frac{d\sigma}{d\Omega} = |f(k,\theta)|^2 \ . \tag{23.50}$$

The optical theorem states that

$$\sigma_{\rm tot} = \frac{4\pi}{k}{\rm Im}\, f(k,0) \ , \tag{23.51}$$

and the cross section in the ℓ^{th} partial wave is therefore bounded:

$$\sigma_\ell = \frac{4\pi}{k^2}(2\ell+1)|a_\ell|^2 \le \frac{4\pi(2\ell+1)}{k^2} \ . \tag{23.52}$$

The evolution with energy of partial-wave amplitude a_ℓ can be displayed as a trajectory in an Argand plot, as shown in Fig. 23.6.

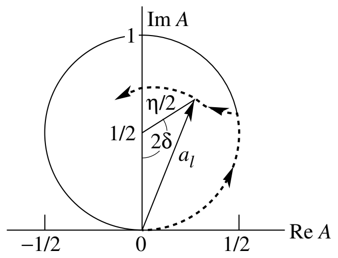

Figure 23.6: Argand plot for the display of a partial-wave amplitude as a function of energy.

The usual Lorentz-invariant matrix element $\mathscr{M}$ (see Sec. 23.3 above) for the elastic process is related to $f(k,\theta)$ by

$$\mathscr{M} = -8\pi\sqrt{s}\, f(k,\theta) \ , \tag{23.53}$$

so

$$\sigma_{\rm tot} = -\frac{1}{2k\sqrt{s}}\ {\rm Im}\,\mathscr{M}(t=0) \ , \tag{23.54}$$

where s and t are the center-of-mass energy squared and momentum transfer squared, respectively (see Sec. 23.4.1).

23.5.3.1. *Resonances:* The Breit-Wigner form for an elastic amplitude a_ℓ with a resonance at c.m. energy E_R, elastic width $\Gamma_{\rm el}$, and total width $\Gamma_{\rm tot}$ is

$$a_\ell = \frac{\Gamma_{\rm el}/2}{E_R - E - i\Gamma_{\rm tot}/2} \ , \tag{23.55}$$

where E is the c.m. energy. As shown in Fig. 23.7, in the absence of background the elastic amplitude traces a counterclockwise circle with center $ix_{\rm el}/2$ and radius $x_{\rm el}/2$, where the elasticity $x_{\rm el} = \Gamma_{\rm el}/\Gamma_{\rm tot}$. The amplitude has a pole at $E = E_R - i\Gamma_{\rm tot}/2$.

The Breit-Wigner cross section for a spin-J resonance produced in the collision of particles of spin S_1 and S_2 is

$$\sigma_{BW}(E) = \frac{(2J+1)}{(2S_1+1)(2S_2+1)}\ \frac{\pi}{k^2}\ \frac{B_{\rm in}B_{\rm out}\Gamma^2_{\rm tot}}{(E-E_R)^2 + \Gamma^2_{\rm tot}/4} \ , \tag{23.56}$$

where k is the c.m. momentum, E is the c.m. energy, and $B_{\rm in}$ and $B_{\rm out}$ are the branching fractions of the resonance into the entrance and exit channels. The $2S+1$ factors are the multiplicities of the incident spin states, so they are replaced by 2 for photons, *etc.* This expression is valid only for a particle of narrow width. If the width

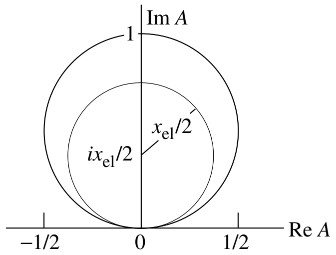

Figure 23.7: Argand plot for a resonance.

is not small, $\Gamma_{\rm tot}$ cannot be treated as a constant independent of E. There are many other forms for σ_{BW}, all of which are equivalent to the one given here in the narrow-width case. Some of these forms may be more appropriate if the resonance is broad.

The relativistic Breit-Wigner form corresponding to Eq. (23.55) is:

$$a_\ell = \frac{-m\Gamma_{\rm el}}{s - m^2 + im\Gamma_{\rm tot}} \ . \tag{23.57}$$

A better form incorporates the known kinematic dependences, replacing $m\Gamma_{\rm tot}$ by $\sqrt{s}\,\Gamma_{\rm tot}(s)$, where $\Gamma_{\rm tot}(s)$ is the width the resonance particle would have if its mass were $\sqrt{s}$, and correspondingly $m\Gamma_{\rm el}$ by $\sqrt{s}\,\Gamma_{\rm el}(s)$ where $\Gamma_{\rm el}(s)$ is the partial width in the incident channel for a mass $\sqrt{s}$:

$$a_\ell = \frac{-\sqrt{s}\,\Gamma_{\rm el}(s)}{s - m^2 + i\sqrt{s}\,\Gamma_{\rm tot}(s)} \ . \tag{23.58}$$

For the Z boson, all the decays are to particles whose masses are small enough to be ignored, so on dimensional grounds $\Gamma_{\rm tot}(s) = \sqrt{s}\,\Gamma_0/m_Z$, where Γ_0 defines the width of the Z, and $\Gamma_{\rm el}(s)/\Gamma_{\rm tot}(s)$ is constant. A full treatment of the line shape requires consideration of dynamics, not just kinematics. For the Z this is done by calculating the radiative corrections in the Standard Model.