24. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

24.1. Leptoproduction

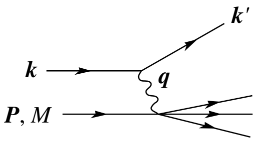

Figure 24.1: Kinematic quantities for description of lepton-nucleon scattering. k and k' are the four-momenta of incoming and outgoing leptons, P is the four-momentum of a nucleon with mass M. The exchanged particle is a γ, $W^{\pm}$, or Z^0; it transfers four-momentum $q = k - k'$ to the target.

Invariant quantities:

$\nu = \dfrac{q \cdot P}{M} = E - E'$ is the lepton's energy loss in the lab (in earlier literature sometimes $\nu = q \cdot P$). Here, E and E' are the initial and final lepton energies in the lab.

$Q^2 = -q^2 = 2(EE' - \overrightarrow{k} \cdot \overrightarrow{k}{}') - m_\ell^2 - m_{\ell'}^2$ where $m_\ell (m_{\ell'})$ is the initial (final) lepton mass. If $EE' \sin^2(\theta/2) \gg m_\ell^2, m_{\ell'}^2$, then

$\approx 4EE' \sin^2(\theta/2)$, where θ is the lepton's scattering angle in the lab.

$x = \dfrac{Q^2}{2M\nu}$ In the parton model, x is the fraction of the target nucleon's momentum carried by the struck quark. See section on QCD.

$y = \dfrac{q \cdot P}{k \cdot P} = \dfrac{\nu}{E}$ is the fraction of the lepton's energy lost in the lab.

$W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$ is the mass squared of the system recoiling against the lepton.

$s = (k + P)^2 = \dfrac{Q^2}{xy} + M^2$

24.1.1. *Leptoproduction cross sections*:

$$\frac{d^2\sigma}{dx\, dy} = \nu\,(s - M^2)\,\frac{d^2\sigma}{d\nu\, dQ^2} = \frac{2\pi\, M\nu}{E'}\,\frac{d^2\sigma}{d\Omega_{\rm lab}\, dE'}$$
$$= x(s - M^2)\,\frac{d^2\sigma}{dx\, dQ^2}\ . \qquad (24.1)$$

24.1.2. *Electroproduction structure functions*:

The neutral-current process, $eN \to eX$, is parity conserving at low Q^2 and can be written in terms of two structure functions $F_1^{\rm NC}(x, Q^2)$ and $F_2^{\rm NC}(x, Q^2)$:

$$\frac{d^2\sigma}{dx\, dy} = \frac{4\pi\, \alpha^2 (s - M^2)}{Q^4}$$
$$\times \left[(1 - y)\, F_2^{\rm NC} + y^2\, xF_1^{\rm NC} - \frac{M^2}{(s - M^2)}\, xy\, F_2^{\rm NC}\right]\ . \qquad (24.2)$$

The charged-current processes, $e^- N \to \nu X$, $\nu N \to e^- X$, and $\overline{\nu} N \to e^+ X$, are parity violating and can be written in terms of three structure functions $F_1^{\rm CC}(x, Q^2)$, $F_2^{\rm CC}(x, Q^2)$, and $F_3^{\rm CC}(x, Q^2)$:

$$\frac{d^2\sigma}{dx\, dy} = \frac{G_F^2\, (s - M^2)}{2\pi}\,\frac{M_W^4}{(Q^2 + M_W^2)^2} \qquad (24.3)$$
$$\times \left\{\left[1 - y - \frac{M^2 xy}{(s - M^2)}\right] F_2^{\rm CC} + \frac{y^2}{2}\, 2x\, F_1^{\rm CC} \pm (y - \frac{y^2}{2})\, x\, F_3^{\rm CC}\right\}\ ,$$

where the last term is positive for the e^- and ν reactions and negative for $\overline{\nu} N \to e^+ X$.

24.1.3. *The QCD parton model*:

In the QCD parton model, the structure functions defined above can be expressed in terms of parton distribution functions. The quantity $f_i(x, Q^2)dx$ is the probability that a parton of type i (quark, antiquark, or gluon), carries a momentum fraction between x and $x + dx$ of the nucleon's momentum in a frame where the nucleon's momentum is large. For the cross section corresponding to the *neutral-current process* $ep \to eX$, we have for $s \gg M^2$ (in the case where the incoming electron is either left- (L) or right- (R) handed):

$$\frac{d^2\sigma}{dx\, dy} = \frac{\pi\alpha^2}{sx^2\, y^2}\left[\sum_q (x\, f_q\, (x, Q^2) + x\, f_{\overline{q}}\, (x, Q^2)\right]$$
$$\times \left[A_q + (1 - y)^2\, B_q\right]\ . \qquad (24.4)$$

Here the index q refers to a quark flavor (*i.e.*, u, d, s, c, b, or t), and

$$A_q = \left(-q_q + g_{Lq}\, g_{Le}\,\frac{Q^2}{Q^2 + M_Z^2}\right)^2 + \left(-q_q + g_{Rq}\, g_{Re}\,\frac{Q^2}{Q^2 + M_Z^2}\right)^2\ , \qquad (24.5)$$

$$B_q = \left(-q_q + g_{Rq}\, g_{Le}\,\frac{Q^2}{Q^2 + M_Z^2}\right)^2 + \left(-q_q + g_{Lq}\, g_{Re}\,\frac{Q^2}{Q^2 + M_Z^2}\right)^2\ . \qquad (24.6)$$

Here q_q is the charge of flavor q. For a left-handed electron, $g_{Re} = 0$ and $g_{Le} = (-1/2 + \sin^2\theta_W)/(\sin\theta_W \cos\theta_W)$, while for a right-handed one, $g_{Le} = 0$ and $g_{Re} = (\sin^2\theta_W)/(\sin\theta_W \cos\theta_W)$. For the quarks, $g_{Lq} = (T_3 - q_q \sin^2\theta_W)/(\sin\theta_W \cos\theta_W)$, and $g_{Rq} = (-q_q \sin^2\theta_W)\ /\ (\sin\theta_W \cos\theta_W)$.

For neutral-current *neutrino* (*antineutrino*) *scattering*, the same formula applies with g_{Le} replaced by $g_{L\nu} = 1/(2\sin\theta_W \cos\theta_W)$ ($g_{L\overline{\nu}} = 0$) and g_{Re} replaced by $g_{R\nu} = 0$ [$g_{R\overline{\nu}} = -1/(2\sin\theta_W \cos\theta_W)$].

In the case of the *charged-current processes* $e_L^- p \to \nu X$ and $\overline{\nu} p \to e^+ X$, Eq. (24.3) applies with

$$F_2 = 2xF_1 = 2x\Big[f_u(x, Q^2) + f_c(x, Q^2)$$
$$+\ f_t(x, Q^2) + f_{\overline{d}}\,(x, Q^2) + f_{\overline{s}}\,(x, Q^2) + f_{\overline{b}}\,(x, Q^2)\Big]\ , \qquad (24.7)$$

$$F_3 = 2\Big[f_u(x, Q^2) + f_c(x, Q^2)$$
$$+\ f_t(x, Q^2) - f_{\overline{d}}\,(x, Q^2) - f_{\overline{s}}\,(x, Q^2) - f_{\overline{b}}\,(x, Q^2)\Big]\ . \qquad (24.8)$$

For the process $\nu p \to e^- X$:

$$F_2 = 2xF_1 = 2x\Big[f_d(x, Q^2) + f_s(x, Q^2)$$
$$+\ f_b(x, Q^2) + f_{\overline{u}}\,(x, Q^2) + f_{\overline{c}}\,(x, Q^2) + f_{\overline{t}}\,(x, Q^2)\Big]\ , \qquad (24.9)$$

$$F_3 = 2\Big[f_d(x, Q^2) + f_s(x, Q^2)$$
$$+\ f_b(x, Q^2) - f_{\overline{u}}\,(x, Q^2) - f_{\overline{c}}\,(x, Q^2) - f_{\overline{t}}\,(x, Q^2)\Big]\ . \qquad (24.10)$$