24.2. e^+e^- annihilation

For pointlike spin-1/2 fermions in the c.m., the differential cross section for $e^+e^- \to f\overline{f}$ via single photon annihilation is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\,\beta\,\left[1+\cos^2\theta + (1-\beta^2)\sin^2\theta\right] Q_f^2\ , \tag{24.11}$$

where β is the velocity of the final state fermion in the center of mass and Q_f is the charge of the fermion in units of the proton charge. For $\beta \to 1$,

$$\sigma = \frac{4\pi\alpha^2}{3s}\,Q_f^2 = \frac{86.8\,Q_f^2\ nb}{s(\text{GeV}^2)}\ . \tag{24.12}$$

At higher energies the Z^0 (mass M_Z and width Γ_Z) must be included, and the differential cross section for $e^+e^- \to f\overline{f}$ becomes

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\,\beta\,\Big[&Q_f^2[1+\cos^2\theta + (1-\beta^2)\,\sin^2\theta]\\ &-2Q_f\,\chi_1\Big\{VV_f\,[1+\cos^2\theta+(1-\beta^2)\,\sin^2\theta] - 2a_f\,\beta\cos\theta\Big\}\\ &+\chi_2\Big\{V_f^2\,(1+V^2)\,[1+\cos^2\theta+(1-\beta^2)\,\sin^2\theta]\\ &+\beta^2\,a_f^2(1+V^2)\,[1+\cos^2\theta] - 8\beta\,VV_f\,a_f\cos\theta\Big\}\Big]\ ,\end{aligned} \tag{24.13}$$

$$\chi_1 = \frac{1}{16\sin^2\theta_W\cos^2\theta_W}\ \frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2}\ , \tag{24.14}$$

$$\chi_2 = \frac{1}{256\sin^4\theta_W\cos^4\theta_W}\ \frac{s^2}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2}\ , \tag{24.15}$$

$$V = -1 + 4\sin^2\theta_W\ , \tag{24.16}$$

$$a_f = 2T_{3f}\ , \tag{24.17}$$

$$V_f = 2T_{3f} - 4Q_f\sin^2\theta_W\ , \tag{24.18}$$

where the subscript f refers to the particular fermion and

$$T_3 = +1/2 \quad \text{for } \nu_e,\, \nu_\mu,\, \nu_\tau,\, u,\, c,\, t, \tag{24.19a}$$

$$T_3 = -1/2 \quad \text{for } e^-,\, \mu^-,\, \tau^-,\, d,\, s,\, b. \tag{24.19b}$$

24.3. Two-photon process at e^+e^- colliders

When an e^+ and an e^- collide with energies E_1 and E_2, they emit dn_1 and dn_2 virtual photons with energies ω_1 and ω_2 and 4-momenta q_1 and q_2. In the equivalent photon approximation, the cross section for $e^+e^- \to e^+e^-X$ is related to the cross section for $\gamma\gamma \to X$ by (Ref. 1)

$$d\sigma_{e^+e^-\to e^+e^-X}\,(s) = dn_1\,dn_2\,d\sigma_{\gamma\gamma\to X}\,(W^2) \tag{24.20}$$

where $s = 4E_1E_2$, $W^2 = 4\omega_1\omega_2$ and

$$dn_i = \frac{\alpha}{\pi}\left[1 - \frac{\omega_i}{E_i} + \frac{\omega_i^2}{2E_i^2} - \frac{m_e^2\omega_i^2}{(-q_i^2)E_i^2}\right]\frac{d\omega_i}{\omega_i}\,\frac{d(-q_i^2)}{(-q_i^2)}\ . \tag{24.21}$$

After integration (including that over q_i^2 in the region $m_e^2\omega_i^2/E_i(E_i-\omega_i) \le -q_i^2 \le (-q^2)_{\max}$), the cross section is

$$\begin{aligned}\sigma_{e^+e^-\to e^+e^-X}\,(s) = \frac{\alpha^2}{\pi^2}\int_{z_{th}}^1 \frac{dz}{z}\Big[&f(z)\left(\ln\frac{(-q^2)_{\max}}{m_e^2 z} - 1\right)^2\\ &-\tfrac{1}{3}\left(\ln\frac{1}{z}\right)^3\Big]\sigma_{\gamma\gamma\to X}\,(zs)\ ;\\ f(z) = \left(1+\tfrac{1}{2}z\right)^2\ln\frac{1}{z} - \tfrac{1}{2}&(1-z)(3+z)\ ;\\ z = \frac{W^2}{s}\ .&\end{aligned} \tag{24.22}$$

The quantity $(-q^2)_{\max}$ depends on properties of the produced system X, in particular, $(-q^2)_{\max} \sim m_\rho^2$ for hadron production ($X = h$) and $(-q^2)_{\max} \sim W^2$ for lepton pair production ($X = \ell^+\ell^-$, $\ell = e,\, \mu,\, \tau$).

For production of a resonance of mass m_R and spin $J \neq 1$

$$\begin{aligned}\sigma_{e^+e^-\to e^+e^-R}\,(s) &= (2J+1)\frac{8\alpha^2\Gamma_{R\to\gamma\gamma}}{m_R^3}\\ &\times\left[f(m_R^2/s)\left(\ln\frac{sm_V^2}{m_e^2m_R^2} - 1\right)^2 - \frac{1}{3}\left(\ln\frac{s}{m_R^2}\right)^3\right]\end{aligned} \tag{24.23}$$

where m_V is the mass that enters into the form factor of the $\gamma\gamma \to R$ transition: $m_V \sim m_\rho$ for $R = \pi^0,\, \rho^0,\, \omega,\, \phi,\, \ldots$, $m_V \sim m_R$ for $R = c\overline{c}$ or $b\overline{b}$ resonances.

24.4. Inclusive hadronic reactions

One-particle inclusive cross sections $Ed^3\sigma/d^3p$ for the production of a particle of momentum p are conveniently expressed in terms of rapidity (see above) and the momentum p_T transverse to the beam direction (defined in the center-of-mass frame)

$$E\frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi\,dy\,p_T\,dp_T}\ . \tag{24.24}$$

In the case of processes where p_T is large or the mass of the produced particle is large (here large means greater than 10 GeV), the parton model can be used to calculate the rate. Symbolically

$$\sigma_{\text{hadronic}} = \sum_{ij}\int f_i(x_1,\,Q^2)\,f_j(x_2,\,Q^2)\,dx_1\,dx_2\,\widehat{\sigma}_{\text{partonic}}\ , \tag{24.25}$$

where $f_i(x,\,Q^2)$ is the parton distribution introduced above and Q is a typical momentum transfer in the partonic process and $\widehat{\sigma}$ is the partonic cross section. Two examples will help to clarify. The production of a W^+ in pp reactions at rapidity y in the center-of-mass frame is given by

$$\begin{aligned}\frac{d\sigma}{dy} &= \frac{G_F\,\pi\sqrt{2}}{3}\\ &\times\tau\Big[\cos^2\theta_c\Big(u(x_1\,,\,M_W^2)\,\overline{d}\,(x_2,M_W^2)\\ &\qquad + u(x_2\,,\,M_W^2)\,\overline{d}\,(x_1,M_W^2)\Big)\\ &\quad+\sin^2\theta_c\Big(u(x_1\,,\,M_W^2)\,\overline{s}\,(x_2\,,\,M_W^2)\\ &\qquad + s(x_2,M_W^2)\,\overline{u}\,(x_1,M_W^2)\Big)\Big]\ ,\end{aligned} \tag{24.26}$$

where $x_1 = \sqrt{\tau}\,e^y$, $x_2 = \sqrt{\tau}\,e^{-y}$, and $\tau = M_W^2/s$. Similarly the production of a jet in pp (or $p\overline{p}$) collisions is given by

$$\begin{aligned}\frac{d^3\sigma}{d^2p_T\,dy} &= \sum_{ij}\int f_i(x_1\,,\,p_T^2)\,f_j(x_2,p_T^2)\\ &\times\quad\left[\widehat{s}\,\frac{d\widehat{\sigma}}{d\,\widehat{t}}\right]_{ij} dx_1\,dx_2\,\delta(\,\widehat{s}+\widehat{t}+\widehat{u})\ ,\end{aligned} \tag{24.27}$$