masses [13]. At low Q^2 values there are also important "higher-twist" (HT) or nonperturbative contributions of the form:

$$F_i(x,Q^2) = F_i^{(LT)}(x,Q^2) + \frac{F_i^{(HT)}(x,Q^2)}{Q^2} + \cdots . \tag{25.13}$$

Leading twist (LT) indicates a term whose behavior is predicted by perturbative QCD. These corrections are numerically important only for $Q^2 < \mathcal{O}(10\,\mathrm{GeV}^2)$ except for x very close to 1.

A detailed review of the current status of the experimental data can be found, for example, in [14], [15] and only a brief summary will be presented here. Since the last version of this *Review* [16] appeared, the discrepancies between different experiments have mostly been resolved. We shall only include determinations of Λ from the recently published results; the earlier editions of this *Review* should be consulted for the earlier data. In any event, the recent results will dominate the average since their errors are smaller.

From Eq. (25.9), it is clear that a nonsinglet structure function offers in principle the most precise test of the theory, since the Q^2 evolution is independent of the unmeasured gluon distribution. The CCFRR collaboration fit to the Gross-Llewellyn Smith sum rule [17] is known to order α_s^3 [18]

$$\int_0^1 dx(F_3^{\overline{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2)) =$$

$$3\left[(1 - \frac{\alpha_s}{\pi}(1 + 3.58\frac{\alpha_s}{\pi} + 19.0(\frac{\alpha_s}{\pi})^2) - \Delta HT\right] . \tag{25.14}$$

Where the higher-twist contribution $\Delta HT = 0.032 \pm 0.016$ [18] Using the CCFRR data [19] this gives $\alpha_s(1.73\ \mathrm{GeV}) = 0.320 \pm 0.043(\mathrm{expt.}) \pm 0.029(\mathrm{theory})$. The error from higher-twist terms dominates the theoretical error, the higher-twist term being approximately 50% larger than the α_s^3 term. Recently a measurement of Λ has been made using F_3 in neutrino scattering [20]. The result is $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 179 \pm 36 \pm 41$ MeV. The errors are statistical and systematic but do not include (theoretical) errors arising from the choice of μ^2. Measurements involving singlet-dominated structure functions such as F_2 result in correlated measurements of $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$ and the gluon distribution. By utilizing high-statistics data at large x (> 0.25) and large Q^2, where F_2 behaves like an nonsinglet and F_3 at smaller x a nonsinglet fit can be performed with better statistical precision and hence the error on the measured value of $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$ is much reduced. CCFRR gives $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 210 \pm 28 \pm 41$ MeV [20] from $F_2(\nu N)$ and $F_3(\nu N)$. There is an additional uncertainty of ± 59 MeV from the choice of scale. The NMC collaboration [21] gives $\alpha_s(7\ \mathrm{GeV}^2) = 0.264 \pm 0.018(\mathrm{stat.}) \pm 0.070(\mathrm{syst.}) \pm 0.013(\text{ higher-twist})$. The systematic error is larger than the CCFRR result, partially because the data are at smaller values of x and the gluon distribution is more important. A reanalysis [22] of EMC data [23] gives $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 211 \pm 80 \pm 80$ MeV from $F_2(\nu N)$. Finally a combined analysis [24] of SLAC [25] and BCDMS [26] data gives $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 263 \pm 42 \pm 55$ MeV. Here the systematic error is an estimate of the uncertainty due to the choice of Q^2 used in the argument of α_s and in the scale at which the structure functions (factorization scale) used in the QCD calculation are evaluated.

The results from [20–22] and [24] can be combined to give $\alpha_s(M_Z) = 0.112 \pm 0.002 \pm 0.004$ or equivalently $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 234 \pm 26 \pm 50$ MeV. Here the former error is a combination of statistical and systematic errors and the second error is due to the scale uncertainty. This result is an average of the results weighted by their statistical and systematic errors. The scale error which is common to all is then reapplied to the average.

Typically, Λ is extracted from the data by parametrizing the parton densities in a simple analytic way at some Q_0^2, evolving to higher Q^2 using the next-to-leading-order evolution equations, and fitting globally to the measured structure functions to obtain $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$. Thus an important by-product of such studies is the extraction of parton densities at a fixed-reference value of Q_0^2. These can then be evolved in Q^2 and used as input for phenomenological studies in hadron-hadron collisions (see below). To avoid having to evolve from the starting Q_0^2 value each time, a parton density is required; it is useful to have available a simple analytic approximation to the densities valid over a range of x and Q^2 values. A package is available in the from the CERN computer library that includes an exhaustive set of fits [27]. Some of these fits are obsolete. In using a parameterization to predict event rates, a next-to-leading order fit must be used if the process being calculated is known to next-to-leading order in QCD perturbation theory. In such a case there is an additional scheme dependence; this scheme dependence is reflected in the $\mathcal{O}(\alpha_s)$ corrections that appear in the relations between the structure functions and the quark distribution functions. There are two common schemes, a deep-inelastic scheme where there are no order α_s corrections in the formula for $F_2(x,Q^2)$ and the minimal subtraction scheme. It is important, when these next-to-leading order fits are used in other processes (see below), that the same scheme is used in the calculation of the partonic rates.

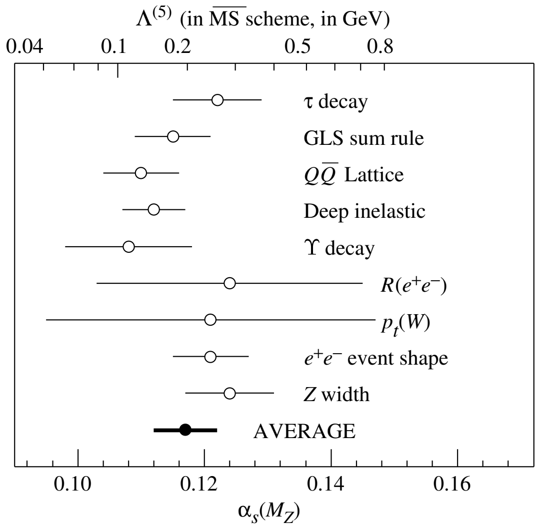

Figure 25.1: Summary of the values of $\alpha_s(M_Z)$ and $\Lambda^{(5)}$ from various processes ordered from top to bottom by increasing energy scale of the measurements. The values shown indicate the process and the measured value of α_s extrapolated up to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties. The weighted average is obtained from the above values using the method discussed in the text.

25.4. QCD in decays of the tau lepton

The semi-leptonic branching ratio of the tau (R_τ) is an inclusive quantity. It is related to the contribution of hadrons to the imaginary part of the W self energy $(\Pi(s))$, just as R (see below, Sec. 25.7) is related to the imaginary part of the photon self energy. However it is more inclusive than R since it involves an integral

$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}(1 - \frac{s}{m_\tau^2})^2 \mathrm{Im}(\Pi(s)) .$$

Since the scale involved is low, one must take into account nonperturbative (higher-twist) contributions which are suppressed by powers of the tau mass.

$$R_\tau = 3.058\left[1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.2\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^2 + 26.4\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^3 \right.$$
$$\left. + a\frac{m^2}{m_\tau^2} + b\frac{m\psi\overline{\psi}}{m_\tau^4} + c\frac{\psi\overline{\psi}\psi\overline{\psi}}{m_\tau^6} + \cdots\right]$$

Here a, b, and c are dimensionless constants and m is a light quark mass. The term of order $1/m_\tau^2$ is a kinematical effect due to the light