quark masses and is consequently very small. The nonperturbative terms are estimated using sum rules [28]. In total they are estimated to be -0.007 ± 0.004 [29]. This estimate relies on there being no term of order Λ^2/m_τ^2 (note that $\frac{\alpha_s(m_\tau)}{\pi} \sim (\frac{0.5 \text{ GeV}}{m_\tau})^2$). The a, b and c can be determined from the data [30] by fitting to moments of the $\Pi(s)$. The values so extracted [31] are consistent with the theoretical estimates. If the nonperturbative terms are omitted from the fit, the extracted value of $\alpha_s(m_\tau)$ decreases by ~ 0.02.

For $\alpha_s(m_\tau) = 0.36$ the perturbative series for R_τ is $R_\tau \sim 3.058(1 + 0.114 + 0.073 + 0.043)$. The size (estimated error) of the nonperturbative term is 20% (7%) of the size of the order α_s^3 term. The perturbation series in not very well convergent; if the order α_s^3 term is omitted the extracted value of $\alpha_s(m_\tau)$ increases by 0.05. Using the experimental average [32] for R_τ of 3.6174 ± 0.034 gives $\alpha_s(m_\tau) = 0.360 \pm 0.031$ using the experimental error alone. We assign a theoretical error equal to 1/2 of the contribution from the order α^3 and nonperturbative contributions. This then gives $\alpha_s(m_\tau) = 0.360 \pm 0.041$ for the final result. Note that the experimental errors are dominant. The small theoretical errors have been criticized [33]. Here it is claimed that the presence of hadronic resonances limits the applicability of perturbative QCD and that the theoretical errors are underestimated by at least a factor of 2. If this argument is correct the agreement with values of α_s from other processes is accidental.

25.5. QCD in high-energy hadron collisions

There are many ways in which perturbative QCD can be tested in high-energy hadron colliders. The quantitative tests are only useful if the process in question has been calculated beyond leading order in QCD perturbation theory. The production of hadrons with large transverse momentum in hadron-hadron collisions provides a direct probe of the scattering of quarks and gluons: $qq \to qq$, $qg \to qg$, $gg \to gg$, *etc.* The present generation of $p\overline{p}$ colliders provide center-of-mass energies which are sufficiently high that these processes can be unambiguously identified in two-jet production at large transverse momentum. Recent higher–order QCD calculations of the jet rates [34] and shapes are in impressive agreement with data [35]. As an example, Fig. 33.5 in this *Review* shows the inclusive jet cross section at zero pseudorapidity as a function of the jet transverse momentum for $p\overline{p}$ collisions. Data are also available on the angular distribution of jets; these are also in agreement with QCD expectations [36,37].

QCD corrections to Drell-Yan type cross sections (*i.e.*, the production in hadron collisions by quark-antiquark annihilation of lepton pairs of invariant mass Q from virtual photons, or of real W or Z bosons) are known [38]. These $\mathcal{O}(\alpha_s)$ QCD corrections are sizable at small values of Q.

It is interesting to note that the corresponding correction to W and Z production, as measured in $p\overline{p}$ collisions at $\sqrt{s} = 0.63$ TeV and $\sqrt{s} = 1.8$ TeV has essentially the same theoretical form and is of order 30%.

The production of W and Z bosons and photons at large transverse momentum can also be used to determine α_s. The leading-order QCD subprocesses are $q\overline{q} \to \gamma g$ and $qg \to \gamma q$. If the parton distributions are taken from other processes and a value of $\Lambda^{(4)}_{\overline{\rm MS}}$ assumed, then an absolute prediction is obtained. Conversely, the data can be used to extract information on quark and gluon distributions and on the value of $\Lambda^{(4)}_{\overline{\rm MS}}$. The next-to-leading-order QCD corrections corrections are known [39,40] (for photons) and for W/Z production [41], and so a precision test is possible in principle. The UA2 collaboration [42] has extracted a value of $\alpha_s(M_W) = 0.123 \pm 0.018(\text{stat.}) \pm 0.017(\text{syst.})$ from the measured ratio $R_W = \frac{\sigma(W + 1jet)}{\sigma(W + 0jet)}$. The result depends on the algorithm used to define a jet, and the dominant systematic errors due to fragmentation and corrections for underlying events are connected to the algorithm (the former causes jet energy to be lost, the latter causes it to be increased). The scale at which $\alpha_s(M)$ is to be evaluated is not clear. A change from $M = M_W$ to $M = M_W/2$ causes a shift of 0.01 in the extracted α_s. The quoted error has been increased to take this into account.

25.6. QCD in heavy quarkonium decay

Under the assumption that the hadronic and leptonic decay widths of heavy $Q\overline{Q}$ resonances can be factorized into a nonperturbative part—dependent on the confining potential—and a calculable perturbative part, the ratios of partial decay widths allow measurements of α_s at the heavy quark mass scale. The most precise data come from the decay widths of the 1^{--} $J/\psi(1S)$ and Υ resonances. The total decay width of the Υ is predicted by perturbative QCD [43]

$$R_\Upsilon = \frac{\Gamma(\Upsilon \to \text{hadrons})}{\Gamma(\Upsilon \to \mu^+\mu^-)}$$
$$= \frac{10(\pi^2 - 9)\alpha_s^3(M)}{9\pi\alpha_{\rm em}^2}\left[1 + \frac{\alpha_s}{\pi}\left(-19.4 + \frac{3\beta_0}{2}\left(1.162 + \ln\left(\frac{2M}{M_\Upsilon}\right)\right)\right)\right]$$

Data are available for the Υ, Υ', Υ'' and ψ. The result is very sensitive to α_s and the data are sufficiently precise ($R_\mu(\Upsilon) = 32.5 \pm 0.9$) [44] that the theoretical errors will dominate. There are theoretical corrections to this simple formula due to the relativistic nature of the $Q\overline{Q}$ system; $v^2/c^2 \sim 0.1$ for the Υ. They are more severe for the ψ. There are also nonperturbative corrections of the form Λ^2/m_Υ^2; again these are more severe for the ψ. A fit to Υ, Υ', and Υ'' [45] gives $\alpha_s(M_Z) = 0.108 \pm 0.001(\text{expt.})$. The results from each state separately and also from the ψ are consistent with each other. There is an uncertainty of order ± 0.005 from the choice of scale; the error from v^2/c^2 corrections is a little larger. $\alpha_s(M_Z) = 0.108 \pm 0.010$ is a fair representation of the total error including the possibility of nonperturbative corrections.

25.7. Perturbative QCD in e^+e^- collisions

The total cross section for $e^+e^- \to$ hadrons is obtained (at low values of $\sqrt{s}$) by multiplying the muon-pair cross section by the factor $R = 3\Sigma_q e_q^2$. The higher-order QCD corrections to this quantity have been calculated, and the results can be expressed in terms of the factor:

$$R = R^{(0)}\left[1 + \frac{\alpha_s}{\pi} + C_2\left(\frac{\alpha_s}{\pi}\right)^2 + C_3\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right] , \qquad (25.15)$$

where $C_2 = 1.411$ and $C_3 = -12.8$ [46].

$R^{(0)}$ can be obtained from the formula for $d\sigma/d\Omega$ for $e^+e^- \to f\overline{f}$ by integrating over Ω. The formula is given in Sec. 24.2 of this *Review.* This result is strictly only correct in the zero-quark-mass limit. The $\mathcal{O}(\alpha_s)$ corrections are also known for massive quarks [47].

A comparison of the theoretical prediction of Eq. (25.15) (corrected for the b-quark mass) with all the available data at values of $\sqrt{s}$ between 20 and 65 GeV gives [48] $\alpha_s(35 \text{ GeV}) = 0.146 \pm 0.030$. The principal advantage of determining α_s from R in e^+e^- annihilation is that there is no dependence on fragmentation models, jet algorithms, *etc.* The size of the order α_s^3 term is of order 40% of that of the order α_s^2 and 3% of the order α_s. If the order α_s^3 term is not included a fit to the data yields $\alpha_s(34 \text{ GeV}) = 0.142 \pm 0.03$, indicating that the theoretical uncertainty is smaller than the experimental error.

Measurements of the ratio of hadronic to leptonic width of the Z at LEP Γ_h/Γ_μ probe the same quantity as R. Using the average of $\Gamma_h/\Gamma_\mu = 20.781 \pm 0.049$ gives $\alpha_s(M_Z) = 0.124 \pm 0.007$ [49]. There are theoretical errors arising from the values of the top quark and Higgs masses which enter due to electroweak corrections to the Z width and from the choice of scale. The dominant error is the first of these (± 0.005).

An alternative method of determining α_s in e^+e^- annihilation is from measuring quantities that are sensitive to the relative rates of two-, three-, and four-jet events. In addition to simply counting jets, there are many possible choices of such "shape variables": thrust [50], energy-energy correlations [51], planar triple-energy correlations [52], average jet mass, *etc.* All of these are infrared safe, which means they can be reliably calculated in perturbation theory. The starting point for all these quantities is the multijet cross section. For example, at order α_s, for the process $e^+e^- \to qqg$:

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} , \qquad (25.16)$$

where