$$x_i = \frac{2E_i}{\sqrt{s}} \tag{25.17}$$

are the center-of-mass energy fractions of the final-state (massless) quarks. A distribution in a "three-jet" variable, such as those listed above, is obtained by integrating this differential cross section over an appropriate phase space region for a fixed value of the variable. The order α_s^2 corrections to this process have been computed as well as the 4-jet final states such as $e^+e^- \to qqgg$.

There are many methods used by the LEP groups [53–56] to determine α_s from the event topology. The jet-counting algorithm originally introduced by the JADE collaboration [57] has been used by the LEP groups. Here particles of momenta p_i and p_j are combined into a pseudo-particle of momentum $p_i + p_j$ if the invariant mass of the pair is less than $y_0\sqrt{s}$. The process is then iterated until no more pairs of particles or pseudo-particles remain. The remaining number is then defined to be the number of jets in the event and can be compared to the QCD prediction.

There are theoretical ambiguities in the way that this process is carried out; quarks and gluons are massless whereas the observed hadrons are not. So that the massive jets that result from this scheme cannot be compared directly to the massless jets of perturbative QCD. Different recombination schemes have been tried, for example combining 3-momenta and then rescaling the energy of the cluster so that it remains massless, and these result in the same data giving a slightly different value [58] of α_s. These differences can be used to determine a systematic error. In addition, since what is observed is hadrons rather than quarks and gluons, a model is needed to describe the evolution of a partonic final state into one involving hadrons so that detector corrections can be applied. The second-order matrix elements are combined with a parton-fragmentation model. This model can then be used to correct the data for a direct comparison with the parton calculation. The different hadronization models that are used [59–62] model the dynamics that are controlled by nonperturbative QCD effects which we cannot yet calculate. The fragmentation parameters of these Monte Carlos are tuned to get agreement with the observed data. The differences between these models contribute to the systematic errors. The systematic errors from recombination schemes and fragmentation effects dominate over the statistical and other errors of the LEP experiments.

The scale M at which $\alpha_s(M)$ is to be evaluated is not clear. The invariant mass of a typical jet (or $\sqrt{sy_0}$) is probably a more appropriate choice than the e^+e^- center of mass energy. If the value is allowed to float in the fit to the data, the data tend to prefer values of order $\sqrt{s}/10$ [63]; the exact value depends on the variable that is fitted. The dominant uncertainties arise from the choice of M and from the freedom in the fragmentation Monte Carlos. An average of many variables and all the LEP experiments [65] gives $\alpha_s(M_Z) = 0.119 \pm 0.006$. The dominant error is theoretical. The situation might improve if complete perturbative calculations to order α_s^3 became available.

The perturbative QCD formulae can break down in special kinematical configurations. For example, the thrust distribution contains terms of the type $\alpha_s \ln^2(1-T)$. The higher orders in the perturbation expansion contains terms of order $\alpha_s^n \ln^m(1-T)$. For $T \sim 1$ (the region populated by 2-jet events) the perturbation expansion is unreliable. The terms with $n \leq m$ can be summed to all orders in α_s [66]. The resummed results give better agreement with the data at large values of T [67]. Such resummed results are not available for all of the shape variables [69], but fits using those available yields a LEP average of $\alpha_s(M_Z) = 0.124 \pm 0.006$ [65]. Some caution should be exercised in using these resummed results because of the possibility of overcounting; the showering Monte Carlos that are used for the fragmentation corrections also generate some of these leading log corrections. The errors in the values of $\alpha_s(M_Z)$ from these shape variables are totally dominated by the theoretical uncertainties associated with the choice of scale and the effects of hadronization Monte Carlos. It is gratifying that the shift from the result using the unresummed formulae is less than the error. Since the data fitted to the resummed and not-resummed formulae are the same. We will use the average of $\alpha_s(M_Z) = 0.121 \pm 0.006$. Since the error is dominantly theoretical, it does not reduce in the average.

Similar studies on event shapes have been undertaken at TRISTAN and at PEP/PETRA. A combined result from various shape parameters by the TOPAZ collaboration gives $\alpha_s(58\ \text{GeV}) = 0.125 \pm 0.009$ using the fixed order QCD result and $\alpha_s(58\ \text{GeV}) = 0.132 \pm 0.008$ using the resummed result [70]. We average these and use $\alpha_s(58\ \text{GeV}) = 0.128 \pm 0.009$. The AMY group has fitted a next to leading order QCD Monte Carlo [68] to the single particle momentum distribution inside jets. The analysis is analogous to the fitting of moments in deep-inelastic scattering. They quote $\alpha_s(58\ \text{GeV}) = 0.134 \pm 0.006$ [71]. Jets are selected so that they are dominantly quark jets since the Monte Carlo does not well describe gluon jets. No systematic error is included for the contribution of gluon jets to the result. The errors could be understated therefore and we have chosen to use $\alpha_s(58\ \text{GeV}) = 0.134 \pm 0.008$ in order to be conservative.

The measurements of event shapes at PEP/PETRA are summarized in earlier editions of this note. The results are consistent with those from Z decay, but have larger errors. We use $\alpha_s(34\ \text{GeV}) = 0.14 \pm 0.02$ [72] Since the errors in the event shape measurements are dominantly systematic, the results from PEP/PETRA, TRISTAN and LEP are combined to give $\alpha_s(M_Z) = 0.121 \pm 0.006$. This result is then used in forming the final average value of α_s.

There are many other ways in which QCD can be tested in electron-positron collisions. Mention should be made in particular of the interesting and important results from "two-photon" processes. See the previous edition of this *Review* [16] for more information. There are no new results and the data do not contribute significantly to the average.

25.8. Lattice QCD

Lattice gauge theory calculations can be used to calculated the energy levels of a $Q\overline{Q}$ system and then extract α_s. The FNAL group [73] uses the splitting between the $1S$ and $1P$ in the charmonium system $[m_{h_c} - (3m_\psi + m_{\eta_c})/4 = 456.6 \pm 0.4\ \text{MeV}]$. to determine α_s. The result quoted is $\alpha_s(M_Z) = 0.108 \pm 0.006$. The splitting is almost independent of the charm-quark mass and is therefore dependent only on Λ. The calculation does not rely on perturbation theory or on nonrelativistic approximation. The main errors are systematic associated with the finite lattice spacing (a), the matching to the perturbatively defined α_s, and quenched approximation used in the calculation. The extrapolation to zero-lattice spacing produces a shift in Λ of order 5% and is therefore quite small. The quenched approximation is more serious. No light quarks are allowed to propagate and hence the extracted value of Λ corresponds to the case of zero flavors. $\alpha_s(M)$ is evolved down from the scale (~ 2.3 GeV) of the lattice used to the scale of momentum transfers appropriate to the charmonium system (~ 700 MeV). The resulting coupling is then evolved back up with the correct number of quark flavors. This produces a shift in $\alpha_s(5\ \text{GeV})$ of order 25%, with a claimed uncertainty of 7%. This error dominates and could be an underestimate as the perturbative running of $\alpha_s(M)$ has to be used at small M. In addition a recent calculation [74] using the strength of the force between two heavy quarks computed in the quenched approximation obtains a value of $\alpha_s(5\ \text{GeV})$ that is consistent with this result. Calculations based on the Υ spectrum using non-relativistic lattice theory give $\alpha_s(M_Z) = 0.112 \pm 0.004$ [75]. Here again the dominant error arises from the quenched approximation. We average the two results since they have a common systematic error and use $\alpha_s(M_Z) = 0.110 \pm 0.006$ in the final average.

25.9. Conclusions

The need for brevity has meant that many other important topics in QCD phenomenology have had to be omitted from this review. One should mention in particular the study of exclusive processes (form factors, elastic scattering, ...), the behavior of quarks and gluons in nuclei, the spin properties of the theory and the importance of polarized scattering data, the interface of soft and hard QCD as manifest, for example, by minijet production and hard diffractive processes, and QCD effects in hadron spectroscopy.

In this short review we have focused on those high-energy processes which currently offer the most quantitative tests of perturbative