(for $\nu_e e$ or $\overline{\nu}_e e$, the charged-current contribution must be included), and

$$-\mathscr{L}^{e\text{Hadron}} = -\frac{G_F}{\sqrt{2}} \times \sum_i \left[C_{1i}\, \overline{e}\, \gamma_\mu\, \gamma^5\, e\, \overline{q}_i\, \gamma^\mu\, q_i + C_{2i}\, \overline{e}\, \gamma_\mu\, e\, \overline{q}_i\, \gamma^\mu\, \gamma^5\, q_i\right] \quad (26.14)$$

(One must add the parity-conserving QED contribution.)

The Standard Model expressions for $\epsilon_{L,R}(i)$, $g^{\nu e}_{V,A}$, and C_{ij} are given in Table 26.2. Note that $g^{\nu e}_{V,A}$ and the other quantities are coefficients of effective four-fermi operators, which differ from the quantities defined in Eq. (26.2) and Eq. (26.3) in the radiative corrections and in the presence of possible physics beyond the Standard Model.

A precise determination of the on-shell s^2_W, which depends only very weakly on m_t and M_H, is obtained from deep inelastic neutrino scattering from (approximately) isoscalar targets. The ratio $R_\nu \equiv \sigma^{NC}_{\nu N}/\sigma^{CC}_{\nu N}$ of neutral- to charged-current cross sections has been measured to 1% accuracy by the CDHS [21] and CHARM [22] collaborations [23,24] at CERN, and the CCFR collaboration at Fermilab [25] has recently reported an even more precise result, so it is important to obtain theoretical expressions for R_ν and $R_{\bar\nu} \equiv \sigma^{NC}_{\bar\nu N}/\sigma^{CC}_{\bar\nu N}$ (as functions of $\sin^2\theta_W$) to comparable accuracy. Fortunately, most of the uncertainties from the strong interactions and neutrino spectra cancel in the ratio.

Table 26.2: Standard Model expressions for the neutral-current parameters for ν-hadron, νe, and e-hadron processes. If radiative corrections are ignored, $\rho = \kappa = 1$, $\lambda = 0$. At $\mathscr{O}(\alpha)$ in the on-shell scheme, $\rho^{NC}_{\nu N} = 1.0083$, $\kappa_{\nu N} = 1.0330$, $\lambda_{u_L} = -0.0032$, $\lambda_{d_L} = -0.0026$, and $\lambda_{u_R} = 1/2\,\lambda_{d_R} = 3.6\times10^{-5}$ for $m_t = 169$ GeV, $M_H = 300$ GeV, $M_Z = 91.187$ GeV, and $\langle Q^2\rangle = 20$ GeV^2. For νe scattering, $\kappa_{\nu e} = 1.0332$ and $\rho_{\nu e} = 1.0131$ (at $\langle Q^2\rangle = 0.$). For atomic parity violation, $\rho'_{eq} = 0.9874$ and $\kappa'_{eq} = 1.033$. For the SLAC polarized electron experiment, $\rho'_{eq} = 0.978$, $\kappa'_{eq} = 1.031$, $\rho_{eq} = 1.001$, and $\kappa_{eq} = 1.06$ after incorporating additional QED corrections, while $\lambda_{2u} = -0.013$, $\lambda_{2d} = 0.003$. The dominant m_t dependence is given by $\rho \sim 1 + \rho_t$, while $\kappa \sim 1 + \rho_t/\tan^2\theta_W$ (on-shell) or $\kappa \sim 1(\overline{\text{MS}})$.

Quantity	Standard Model Expression
$\epsilon_L(u)$	$\rho^{NC}_{\nu N}\left(\frac{1}{2} - \frac{2}{3}\kappa_{\nu N}\, \sin^2\theta_W + \lambda_{uL}\right)$
$\epsilon_L(d)$	$\rho^{NC}_{\nu N}\left(-\frac{1}{2} + \frac{1}{3}\kappa_{\nu N}\, \sin^2\theta_W + \lambda_{dL}\right)$
$\epsilon_R(u)$	$\rho^{NC}_{\nu N}\left(-\frac{2}{3}\kappa_{\nu N}\, \sin^2\theta_W + \lambda_{uR}\right)$
$\epsilon_R(d)$	$\rho^{NC}_{\nu N}\left(\frac{1}{3}\kappa_{\nu N}\, \sin^2\theta_W + \lambda_{dR}\right)$
$g^{\nu e}_V$	$\rho_{\nu e}\left(-\frac{1}{2} + 2\kappa_{\nu e}\, \sin^2\theta_W\right)$
$g^{\nu e}_A$	$\rho_{\nu e}\left(-\frac{1}{2}\right)$
C_{1u}	$\rho'_{eq}\left(-\frac{1}{2} + \frac{4}{3}\kappa'_{eq}\, \sin^2\theta_W\right)$
C_{1d}	$\rho'_{eq}\left(\frac{1}{2} - \frac{2}{3}\kappa'_{eq}\, \sin^2\theta_W\right)$
C_{2u}	$\rho_{eq}\left(-\frac{1}{2} + 2\kappa_{eq}\, \sin^2\theta_W\right) + \lambda_{2u}$
C_{2d}	$\rho_{eq}\left(\frac{1}{2} - 2\kappa_{eq}\, \sin^2\theta_W\right) + \lambda_{2d}$

A simple zeroth-order approximation is

$$R_\nu = g^2_L + g^2_R r \quad (26.15a)$$

$$R_{\overline{\nu}} = g^2_L + \frac{g^2_R}{r}\ , \quad (26.15b)$$

where

$$g^2_L \equiv \epsilon_L\,(u)^2 + \epsilon_L\,(d)^2 \approx \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W \quad (26.16a)$$

$$g^2_R \equiv \epsilon_R\,(u)^2 + \epsilon_R\,(d)^2 \approx \frac{5}{9}\sin^4\theta_W\ , \quad (26.16b)$$

and $r \equiv \sigma^{CC}_{\overline{\nu}N}/\sigma^{CC}_{\nu N}$ is the ratio of $\overline{\nu}$ and ν charged-current cross sections, which can be measured directly. [In the simple parton model, ignoring hadron energy cuts, $r \approx (\frac{1}{3} + \epsilon)/(1 + \frac{1}{3}\epsilon)$, where $\epsilon \sim 0.125$ is the ratio of the fraction of the nucleon's momentum carried by antiquarks to that carried by quarks.] In practice, Eq. (26.15) must be corrected for quark mixing, the s and c seas, c-quark threshold effects, nonisoscalar target effects, W-Z propagator differences, and radiative corrections (which lower the extracted value of $\sin^2\theta_W$ by ~ 0.009). Details of the neutrino spectra, experimental cuts, x and Q^2 dependence of structure functions, and longitudinal structure functions enter only at the level of these corrections and therefore lead to very small uncertainties. The largest theoretical uncertainty is associated with the c threshold, which mainly affects σ^{CC}. Using the slow rescaling prescription [9] the central value of $\sin^2\theta_W$ varies as $0.013\ [m_c(\text{GeV})\text{–}1.3]$, where m_c is the effective mass. For $m_c = 1.31 \pm 0.24$ GeV (determined from ν-induced dimuon production [25]) this contributes ±0.003 to the total theoretical uncertainty $\Delta\sin^2\theta_W \sim \pm0.004$. This would require a high-energy neutrino beam for improvement. (The experimental uncertainty is ±0.003). The CCFR group quotes $s^2_W = 0.2218 \pm 0.0059$ for $(m_t, M_H) = (150, 100)$, but this result is insensitive to (m_t, M_H). Combining all of the precise deep-inelastic measurements, one obtains $s^2_W = 0.2260 \pm 0.0048$ for (m_t, M_H) in the allowed range.

The laboratory cross section for $\nu_\mu e \to \nu_\mu e$ or $\overline{\nu}_\mu e \to \overline{\nu}_\mu e$ elastic scattering is

$$\frac{d\sigma_{\nu_\mu,\overline{\nu}_\mu}}{dy} = \frac{G^2_F m_e E_\nu}{2\pi} \times\left[(g^{\nu e}_V \pm g^{\nu e}_A)^2 + (g^{\nu e}_V \mp g^{\nu e}_A)^2(1-y)^2 - (g^{\nu e 2}_V - g^{\nu e 2}_A)\frac{y\, m_e}{E_\nu}\right]\ , \quad (26.17)$$

where the upper (lower) sign refers to $\nu_\mu(\overline{\nu}_\mu)$, and $y \equiv E_e/E_\nu$ [which runs from 0 to $(1 + m_e/2E_\nu)^{-1}$] is the ratio of the kinetic energy of the recoil electron to the incident ν or $\overline{\nu}$ energy. For $E_\nu \gg m_e$ this yields a total cross section

$$\sigma = \frac{G^2_F\, m_e\, E_\nu}{2\pi}\left[(g^{\nu e}_V \pm g^{\nu e}_A)^2 + \frac{1}{3}(g^{\nu e}_V \mp g^{\nu e}_A)^2\right]\ . \quad (26.18)$$

The most accurate leptonic measurements [26–28] of $\sin^2\theta_W$ are from the ratio $R \equiv \sigma_{\nu_\mu e}/\sigma_{\overline{\nu}_\mu e}$ in which many of the systematic uncertainties cancel. Radiative corrections (other than m_t effects) are small compared to the precision of present experiments and have negligible effect on the extracted $\sin^2\theta_W$. The most precise (CHARM II) experiment [28] determined not only $\sin^2\theta_W$ but $g^{\nu e}_{V,A}$ as well. The cross sections for $\nu_e e$ and $\overline{\nu}_e e$ may be obtained from Eq. (26.17) by replacing $g^{\nu e}_{V,A}$ by $g^{\nu e}_{V,A} + 1$, where the 1 is due to the charged-current contribution.

The SLAC polarized-electron experiment [29] measured the parity-violating asymmetry

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}\ , \quad (26.19)$$

where $\sigma_{R,L}$ is the cross section for the deep-inelastic scattering of a right- or left-handed electron: $e_{R,L}N \to e\text{X}$. In the quark parton model

$$\frac{A}{Q^2} = a_1 + a_2\,\frac{1 - (1-y)^2}{1 + (1-y)^2}\ , \quad (26.20)$$