where $Q^2 > 0$ is the momentum transfer and y is the fractional energy transfer from the electron to the hadrons. For the deuteron or other isoscalar target, one has, neglecting the s quark and antiquarks,

$$a_1 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{1u} - \frac{1}{2}C_{1d}\right) \approx \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(-\frac{3}{4} + \frac{5}{3}\sin^2\theta_W\right) \tag{26.21a}$$

$$a_2 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{2u} - \frac{1}{2}C_{2d}\right) \approx \frac{9G_F}{5\sqrt{2}\pi\alpha}\left(\sin^2\theta_W - \frac{1}{4}\right) . \tag{26.21b}$$

Radiative corrections (other than m_t effects) lower the extracted value of $\sin^2\theta_W$ by $\sim$ 0.005.

Experiments measuring atomic parity violation [30] are now quite precise, and the uncertainties associated with atomic wave functions are relatively small (especially for cesium, for which the theoretical uncertainty is $\sim 1\%$ [31]). For heavy atoms one determines the "weak charge"

$$\begin{aligned} Q_W &= -2\left[C_{1u}\,(2Z+N) + C_{1d}(Z+2N)\right] \\ &\approx Z(1-4\sin^2\theta_W) - N \ . \end{aligned} \tag{26.22}$$

Radiative corrections increase the extracted $\sin^2\theta_W$ by ~ 0.008.

The forward-backward asymmetry for $e^+e^- \to \ell\overline{\ell}$, $\ell = \mu$ or τ, is defined as

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \ , \tag{26.23}$$

where $\sigma_F(\sigma_B)$ is the cross section for ℓ^- to travel forward (backward) with respect to the e^- direction. A_{FB} and R, the total cross section relative to pure QED, are given by

$$R = F_1 \tag{26.24}$$

$$A_{FB} = 3F_2/4F_1 \ , \tag{26.25}$$

where

$$F_1 = 1 - 2\chi_0\, g_V^e\, g_V^\ell\, \cos\delta_R + \chi_0^2\left(g_V^{e2} + g_A^{e2}\right)\left(g_V^{\ell 2} + g_A^{\ell 2}\right) \tag{26.26a}$$

$$F_2 = -2\chi_0\, g_A^e\, g_A^\ell\, \cos\delta_R + 4\chi_0^2\, g_A^e\, g_A^\ell\, g_V^e\, g_V^\ell \ , \tag{26.26b}$$

where

$$\tan\delta_R = \frac{M_Z\Gamma_Z}{M_Z^2 - s} \tag{26.27}$$

$$\chi_0 = \frac{G_F}{2\sqrt{2}\pi\alpha}\frac{sM_Z^2}{\left[(M_Z^2 - s)^2 + M_Z^2\Gamma_Z^2\right]^{1/2}} \tag{26.28}$$

and $\sqrt{s}$ is the CM energy. Eq. (26.26) is valid at tree level. If the data are radiatively corrected for QED effects (as described above), then the remaining electroweak corrections can be incorporated [32] (in an approximation adequate for existing PEP, PETRA, and TRISTAN data, which are well below the Z pole) by replacing χ_0 by $\chi(s) \equiv (1+\rho_t)\chi_0(s)\alpha/\widehat{\alpha}(s)$, where $\widehat{\alpha}(s)$ is the running QED coupling, and evaluating g_V in the $\overline{\rm MS}$ scheme. Formulas for $e^+e^- \to hadrons$ may be found in Ref. 33.

At LEP and SLC, there are high-precision measurements of various Z-pole observables. These include the Z mass and total width Γ_Z, and partial widths $\Gamma(f\overline{f})$ for $Z \to f\overline{f}$ for fermion f ($f = e$, μ, τ, hadrons, b, c, and ν). The data is consistent with lepton-family universality $\Gamma(e^+e^-) = \Gamma(\mu^+\mu^-) = \Gamma(\tau^+\tau^-)$, so one may work with an average width $\Gamma(\ell\overline{\ell})$. It is convenient to use the variables M_Z, Γ_Z, $R \equiv \Gamma(\text{had})/\Gamma(\ell\overline{\ell})$, $\sigma_{\text{had}} \equiv 12\pi\Gamma(e^+e^-)\Gamma(\text{had})/M_Z^2\,\Gamma_Z^2$, $R_b \equiv \Gamma(b\overline{b})/\Gamma(\text{had})$, and $R_c \equiv \Gamma(c\overline{c})/\Gamma(\text{had})$, which are weakly correlated experimentally. ($\Gamma(\text{had})$ is the partial width into hadrons.) R is insensitive to m_t (except for $Z \to b\overline{b}$ vertex corrections) and is especially useful for constraining α_s. The width for invisible decays, $\Gamma(\text{inv}) = \Gamma_Z - 3\Gamma(\ell\overline{\ell}) - \Gamma(\text{had}) = 498.2 \pm 4.2$ MeV, can be used to determine the number of neutrino flavors lighter than $M_Z/2$, $N_\nu = \Gamma_{\text{inv}}/\Gamma(\nu\overline{\nu}) \sim 2.98 \pm 0.03$.

There are also measurements of various asymmetries. These include the polarization or left-right asymmetry

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \ , \tag{26.29}$$

where $\sigma_L(\sigma_R)$ is the cross section for a left- (right)-handed incident electron. A_{LR} has been measured precisely by the SLD collaboration at SLC [34] and has the advantages of being extremely sensitive to $\sin^2\theta_W$ and insensitive to QED radiative corrections. Other asymmetries are the forward-backward asymmetries $A_{FB}^{(0,f)}$ for $f = e$, μ, τ, b, c ($A_{FB}^{(0,e)}$, $A_{FB}^{(0,\mu)}$, $A_{FB}^{(0,\tau)}$ are consistent with lepton-family universality, allowing an average value $A_{FB}^{(0,\ell)}$), the jet-charge asymmetry, the τ polarization P_τ, and its angular distribution. Further details, including references to the data from the LEP experiments (ALEPH, DELPHI, L3, OPAL) may be found in the Full Listings in the 'Note on the Z Boson.' At tree level and neglecting QED effects and terms of order $(\Gamma_Z/M_Z)^2$, one has

$$A_{FB}^{(0,f)} \approx \frac{3}{4}A_f\,\frac{A_e + P_e}{1 + P_eA_e} \tag{26.30}$$

$$A_{LR} \approx A_eP_e \ , \tag{26.31}$$

where P_e is the initial e^- polarization and

$$A_f \equiv \frac{2g_V^f\,g_A^f}{g_V^{f2} + g_A^{f2}} \ . \tag{26.32}$$

Similarly, A_τ is given by the negative total τ polarization, and A_e can be extracted from the angular distribution of the polarization.

It has become customary for the experimental groups to present corrected asymmetries A^0, in which photon exchange and γ-Z interference, QED corrections, and corrections for $\sqrt{s} \neq M_Z$ are removed from the data, leaving the pure electroweak asymmetries. Ignoring negligible electroweak boxes, these corrected asymmetries are expressed using effective tree-level expression *e.g.*, $A_{FB}^{(0,f)} = \frac{3}{4}\overline{A}_f\overline{A}_e$ (for $P_e = 0$) and $A_{LR}^0 = \overline{A}_e$, where

$$\overline{A}_f = \frac{2\overline{g}_V^f\,\overline{g}_A^f}{\overline{g}_V^{f2} + \overline{g}_A^{f2}} \ , \tag{26.33a}$$

and

$$\overline{g}_V^f = \sqrt{\rho_f}\;(t_{3L}^{(f)} - 2q_f\kappa_f\sin^2\theta_W) \tag{26.33b}$$

$$\overline{g}_A^f = \sqrt{\rho_f}\;t_{3L}^{(f)} \ . \tag{26.33c}$$

The electroweak-radiative corrections have been absorbed into corrections $\rho_f - 1$ and $\kappa_f - 1$, which depend on the fermion f and on the renormalization scheme. In the on-shell scheme, the quadratic m_t dependence is give by $\rho_f \sim 1 + \rho_t$, $\kappa_f \sim \kappa_f^{os} \sim 1 + \rho_t/\tan^2\theta_W$, while in $\overline{\rm MS}$, $\rho_f \sim \widehat{\rho}$, $\kappa_f \equiv \widehat{\kappa}_f \sim 1$. In practice, additional bosonic loops, vertex corrections, *etc.*, must be included. For example, in the $\overline{\rm MS}$ scheme one has, for $(m_t, M_H) = (169, 300)$, $\rho_\ell = 1.0045$ and $\widehat{\kappa}_\ell = 1.0009$. It is convenient to define an effective angle $\overline{s}_f^2 \equiv \sin^2\overline{\theta}_{Wf} \equiv \widehat{\kappa}_f\widehat{s}_Z^{\,2} = \kappa_f^{os}\,s_W^2$, in terms of which $\overline{g}_V^f$ and $\overline{g}_A^f$ are given by $\sqrt{\rho_f}$ times their tree-level formulae. Because $\overline{g}_V^\ell$ is very small, not only A_{LR}^0 $A_{FB}^{(0,\ell)}$, and P_τ^0, but also $A_{FB}^{(0,b)}$, $A_{FB}^{(0,c)}$, and the jet-charge asymmetry are mainly sensitive to $\overline{s}_\ell^2$. One finds that $\widehat{\kappa}_f$ is almost independent of (m_t, M_H), so that

$$\overline{s}_\ell^2 \sim \widehat{s}_Z^{\,2} + 0.0002 \tag{26.34}$$

using Ref. 15, or $\overline{s}_\ell^2 \sim \widehat{s}_Z^{\,2} + 0.00028$ from Ref. 14 (the small difference is an indication of theoretical uncertainties from higher-order terms, *etc.*). In any case, the asymmetries determine values of $\overline{s}_\ell^2$ and $\widehat{s}_Z^{\,2}$ almost independent of m_t, while the κ's for the other schemes are m_t dependent.