26.3. W and Z decays

The partial decay width for gauge bosons to decay into massless fermions $f_1\overline{f}_2$ is

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 227 \pm 1 \text{ MeV} \tag{26.35a}$$

$$\Gamma(W^+ \to u_i\overline{d}_i) = \frac{C G_F M_W^3}{6\sqrt{2}\pi} |V_{ij}|^2 \approx (707 \pm 3)\, |V_{ij}|^2 \text{ MeV} \tag{26.35b}$$

$$\Gamma(Z \to \psi_i\overline{\psi}_i) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi} \left[g_V^{i2} + g_A^{i2}\right] \tag{26.35c}$$

$$\approx \begin{cases} 167.1 \pm 0.3 \text{ MeV } (\nu\overline{\nu}), & 83.9 \pm 0.2 \text{ MeV } (e^+e^-), \\ 298.0 \pm 0.6 \text{ MeV } (u\overline{u}), & 384.5 \pm 0.8 \text{ MeV } (d\overline{d}), \\ 375.2 \mp 0.4 \text{ MeV } (b\overline{b}). \end{cases}$$

For leptons $C = 1$, while for quarks $C = 3\Big(1+\alpha_s(M_V)/\pi + 1.409\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3\Big)$, where the 3 is due to color and the factor in parentheses is a QCD correction. Corrections to Eq. (26.35) for massive fermions are given in Refs. 10 and 35 and the mass/Yukawa effects in the QCD corrections in Refs. [18,36]. Expressing the widths in terms of $G_F M_{W,Z}^3$ incorporates the bulk of the low-energy radiative corrections [10,35]. The $Z \to f\overline{f}$ widths have an additional QED correction $1 + 3\alpha\, q_f^2/4\pi$. The electroweak corrections are incorporated by replacing $g_{V,A}^{i2}$ by $\overline{g}_{V,A}^{i2}$. Hence, the widths are proportional to $\rho_i \sim 1 + \rho_t$. There is additional (negative) quadratic m_t dependence in the $Z \to b\overline{b}$ vertex corrections [37] which causes $\Gamma(b\overline{b})$ to decrease with m_t. The dominant effect is to multiply $\Gamma(b\overline{b})$ by the vertex correction $1 + \delta\rho_{b\overline{b}}$, where $\delta\rho_{b\overline{b}} \sim 10^{-2}(-\frac{1}{2}\frac{m_t^2}{M_Z^2} + \frac{1}{5})$. In practice, the corrections are included in ρ_b and κ_b.

For 3-fermion families the total widths are predicted to be

$$\Gamma_Z \approx 2.493 \pm 0.004 \text{ GeV} \tag{26.36}$$

$$\Gamma_W \approx 2.09 \pm 0.01 \text{ GeV} \ . \tag{26.37}$$

The numerical values for the widths assume $M_Z = 91.187 \pm 0.007$ GeV, $M_W = 80.29 \pm 0.11$ GeV, $\alpha_s = 0.120$, and $m_t = 169^{+16}_{-18}$ GeV, where the M_W, α_s, and m_t values are predicted by the global fits for $M_H = 300$ GeV. The uncertainties for Γ_W and Γ_Z are dominated by ΔM_W and Δm_t, respectively. The uncertainty in α_s, ± 0.007, introduces an additional uncertainty of 0.2% in the hadronic widths, corresponding to ± 4 MeV in Γ_Z.

These predictions are to be compared with the experimental results $\Gamma_Z = 2.490 \pm 0.007$ GeV and $\Gamma_W = 2.08 \pm 0.07$ GeV.

26.4. Experimental results

The values of the principal Z-pole observables are listed in Table 26.3, along with the Standard Model predictions for $M_Z = 91.187 \pm 0.007$, $m_t = 169^{+16}_{-18}$ GeV (for $M_H = 300$ GeV), 60 GeV $< M_H < 1$ TeV, and $\alpha_s = 0.120 \pm 0.007$. The values and predictions of M_W, M_W/M_Z, the Q_W for cesium [30,31], and recent results from deep inelastic and $\nu_e e$ scattering are also listed. The agreement is generally excellent, although $\Gamma_b = \Gamma(b\overline{b})/\Gamma(\text{had})$ is $\sim 1.8\sigma$ above the Standard Model prediction, and the left-right A_{LR}^0 is 2σ above the Standard Model prediction. There is also an experimental difference of $\sim 2\sigma$ between the SLD value of $A_e^0 = A_{LR}^0$ and the LEP value $A_{\ell\text{LEP}}^0 \sim 0.1434 \pm 0.0073$ obtained from $A_{FB}^{(0,\ell)}$, $A_e^0(P_\tau)$, $A_\tau^0(P_\tau)$ assuming lepton family universality. The observables in Table 26.3 (including correlations on the LEP observables), as well as all low-energy neutral-current data [9], are used in the global fits described below. The parameter $\sin^2\theta_W$ can be determined from the Z-pole observables and M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results [9], shown in Table 26.4, are in impressive agreement with each other, indicating the quantitative success of the Standard Model. The one discrepancy is the value $\widehat{s}_Z^2 = 0.2292 \pm 0.0010$ from A_{LR}^0 which is 2.4σ below the value (0.2319 ± 0.005) from the global fit to all data and 2.8σ below the value 0.2323 ± 0.0005 obtained from all data other than A_{LR}^0.

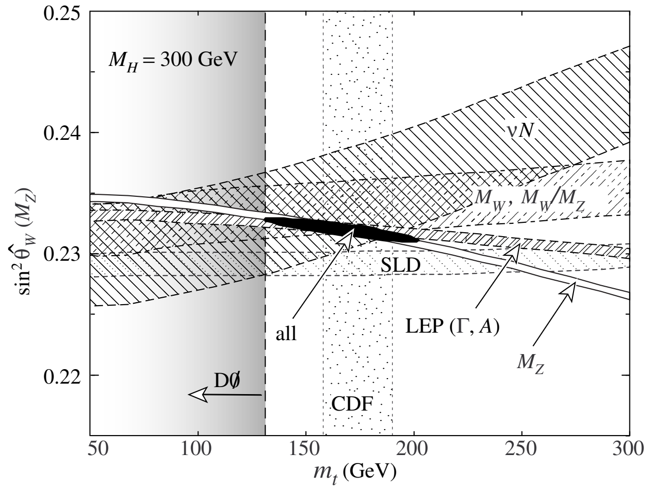

Figure 26.1: One-standard-deviation uncertainties in $\sin^2\widehat{\theta}_W$ as a function of m_t, the direct constraint $m_t > 131$ GeV, the CDF range 174 ± 16 GeV, and the 90% CL region in $\sin^2\widehat{\theta}_W - m_t$ allowed by all data, assuming $M_H = 300$ GeV.

The data allow a simultaneous determination of $\sin^2\theta_W$, m_t, and the strong coupling $\alpha_s(M_Z)$. The latter is determined mainly from Γ_Z and R, and is almost uncorrelated from the other variables. The global fit to all indirect data yields

$$\begin{aligned} \widehat{s}_Z^2 &= 0.2319 \pm 0.0005 \pm 0.0002 \\ m_t &= 169^{+16+17}_{-18-20} \text{ GeV} \\ \alpha_s(M_Z) &= 0.120 \pm 0.007 \pm 0.002 \ , \end{aligned} \tag{26.38}$$

where the central values are for a Higgs mass of 300 GeV, and the second error bars are for $M_H \to 1000(+)$ or $60(-)$ GeV. In all fits, the errors include full statistical, systematic, and theoretical uncertainties. The $\widehat{s}_Z^2$ error is dominated by m_t, and $\widehat{s}_Z^2$ and m_t have a strong negative correlation of ~ -0.85. In the on-shell scheme one has $s_W^2 = 0.2247 \pm 0.0019$, the larger error due to the stronger sensitivity to m_t. The extracted value of α_s is based on a formula which has almost no theoretical uncertainty, and is in excellent agreement with the values 0.121 ± 0.006 from jet-event shapes, at the Z pole, and 0.116 ± 0.005 extracted from low-energy and jet data (see the section on Quantum Chromodynamics). Including the latter value as a separate constraint in the fits has negligible effect on $\sin^2\theta_W$ and m_t.

The value of m_t predicted by the precision data is in remarkable agreement with the value $m_t = 174 \pm 16$ GeV suggested by the CDF candidate top events. (The indirect prediction is for the pole mass, which should correspond approximately to the kinematic mass extracted from the CDF events.) One can carry out a combined fit of the indirect and direct (CDF) data, with the result $m_t = 172^{+11+8}_{-12-9}$ GeV, with little change in the $\sin^2\theta_W$ and α_s values. The results of fits to various combinations of the data are shown in Table 26.5 and the relation between $\widehat{s}_Z^2$ and m_t for various observables in Fig. 26.1.

The data indicate a preference for a small Higgs mass. This is because there is a strong correlation between the quadratic ρ_t terms and logarithmic M_H effects in all of the indirect data except the $Z \to b\overline{b}$ vertex. The latter favor a smaller m_t and therefore a smaller M_H. The difference in χ^2 for the global fit is $\Delta\chi^2 = \chi^2(M_H = 1000 \text{ GeV}) - \chi^2(M_H = 60 \text{ GeV}) = 2.9$. However, this sensitivity is reduced to $\Delta\chi^2 \sim 2.1$ when the direct CDF value of m_t is included. Hence, the data favor a small value of M_H, as in supersymmetric extensions of the Standard Model, and m_t on the lower side of the allowed range; but the effect is marginal statistically: