weak eigenstates to be simultaneously mass eigenstates. However, in the general case of possibly massive (nondegenerate) neutrinos, the weak eigenstates have no well-defined masses, but instead are linear combinations of mass eigenstates. Let us denote the charged leptons as the set $\{\ell_a\}$, $a = 1, \ldots, n$, where $n \geq 3$, with $\ell_1 = e$, $\ell_2 = \mu$, and $\ell_3 = \tau$. From the LEP measurement of the Z width (see section on "Number of Light Neutrinos"), one knows that there are only three neutrinos which couple to the Z in the usual way and have masses $m_\nu < m_Z/2$. Of course, this measurement does not preclude the existence of electroweak-singlet neutrinos. The latter are often called "right-handed neutrino singlets," although, since they are singlets, it is a convention whether one chooses to write them as $(\chi_j)_R$ or $(\chi'_j)_L = (\chi^c_j)_L$. The left-handed components of the weak eigenstates of the neutrinos, $(\nu_{\ell_a})_L$ can be expressed in terms of mass eigenstates by the transformation

$$(\nu_{\ell_a})_L = \sum_j U_{aj} (\nu_j)_L \tag{1}$$

where the $\{\nu_j\}$ denote these mass eigenstates and consist of n members together with possible additional SU(2) × U(1) singlet neutral leptons, often called "sterile" neutrinos. The ordering of the mass eigenbasis can be defined so that U is as nearly diagonal as possible, *i.e.* (with no sum on j) $|U_{jj}| \geq |U_{jk}|$, $k \neq j$. Of course, this does not imply that $m_{\nu_j} > m_{\nu_k}$ for $j > k$.

Thus, as was noted in Ref. 6, decays such as $^3\mathrm{H} \to {}^3\mathrm{He}\, e^- \overline{\nu}_e$ and $\pi^+ \to \mu^+ \nu_\mu$, which have been used to set the best bounds on the respective neutrino masses, really consist of sums of the separate decay modes $^3\mathrm{H} \to {}^3\mathrm{He}\, e^- \overline{\nu}_j$ and $\pi^+ \to \mu^+ \nu_k$, where the ν_j and ν_k are mass eigenstates, and the indices j and k range over all of the values allowed by phase space in these respective decays. The coupling strengths for the j'th mode in ^{3}H β decay and the k'th mode in $\pi^+_{\mu 2}$ decay are given, respectively, by $|U_{1j}|^2$ and $|U_{2k}|^2$. In general, these modes are incoherent, although in the limit in which the ν_j all become degenerate they would become coherent. There are, in addition certain kinematic factors depending on the m_{ν_j} which enter in determining the branching ratio for a given decay mode. Assuming that the off-diagonal elements of the lepton mixing matrix U are small relative to the diagonal elements, the dominantly coupled decays are the ones with coupling strength $|U_{aj}|^2$, $a = j$, *i.e.*, $^3\mathrm{H} \to {}^3\mathrm{He}\, e^- \overline{\nu}_1$ and $\pi^+ \to \mu^+ \nu_2$.

Hence, it follows that the neutrino mass limits quoted in the literature for "m_{ν_e}," "m_{ν_μ}," and "m_{ν_τ}" should really be interpreted as limits on the corresponding mass eigenstates [6,7]. Specifically, a bound on "m_{ν_e}" from a study of tritium β decay, for example, really constitutes a weighted limit on each of the mass eigenstates ν_j in the weak eigenstate ν_e which are kinematically allowed to occur in tritium decay and which are coupled with strength $|U_{1j}|^2$ sufficiently large to make a significant contribution to the observed spectrum. It is thus certainly a limit on ν_1, since this is, by the definition, of the order of the mass eigenbasis, the dominantly coupled neutrino. If lepton mixing is hierarchical, as quark mixing is known to be, *i.e.*, if $|U_{jj}|^2 \gg |U_{jk}|^2$, $j \neq k$, then ν_1 is the only mass eigenstate significantly constrained by a bound on "m_{ν_e}." Furthermore, strictly speaking, a neutrino mass limit cannot be stated in isolation; it always contains some implicit dependence on the relevant lepton-mixing angles. This dependence is fortunately relatively unimportant for the dominantly coupled decay modes, *i.e.*, $e\,\overline{\nu}_1$, $\mu\,\overline{\nu}_2$, and $\tau\,\overline{\nu}_3$ and hence the mass limits on "m_{ν_e}," "m_{ν_μ}," and "m_{ν_τ}" can be reinterpreted as being limits on m_{ν_j}, $j = 1$, 2, and 3, respectively.

There are three general types of (Lorentz-invariant) neutrino mass terms: Dirac masses of the form $m_D\, \overline{\nu}_L\, \chi_R + h.c.$, left-handed Majorana masses of the form $m_L\, \overline{\nu}_L\, \nu^c_R + h.c. = m^*_L\, {\nu_L}^T C \nu_L + h.c.$ and right-handed Majorana masses of the form $m_R\, \overline{\chi}^c_L\, \chi_R + h.c. = m_R\, {\chi_R}^T C\, \chi_R + h.c.$, where C is the Dirac charge conjugation matrix. Dirac mass terms conserve total lepton number L_{tot}, while Majorana mass terms violate L_{tot}. In the standard electroweak theory, extended to include massive neutrinos, (i) a Dirac mass term transforms as a weak $I = 1/2$ operator, and is coupled to the $I = 1/2$ Higgs to make an SU(2) × U(1) singlet operator; (ii) a Majorana mass term involving the $I = 1/2$ left-handed neutrinos transforms as $I = 1$ and must be coupled to an operator with $I = 1$ (and $Y = 2$) to make a gauge-invariant singlet; (iii) a Majorana-mass term involving the SU(2) × U(1) singlet neutral leptons, conventionally considered to be right-handed, is a singlet; it could be present as a bare mass term or couple to some other singlet operator. Note that in the minimal supersymmetric standard model (MSSM), which has two Higgs doublets, of hypercharge $Y = 1$ and $Y = -1$, the Dirac neutrino mass term arises from the cubic chiral superfield coupling $\epsilon_{ij} \widehat{L}^i \widehat{\chi}^c \widehat{H}^j_u$ (all chiral superfields are taken as left-handed), where $\widehat{H}_u$ is the same Higgs that gives mass to the $Q = 2/3$ quarks. The Dirac neutrino mass terms are thus proportional to $\sin\beta$, where $\tan\beta = v_u/v_d$ is the ratio of the vacuum expectation values of the two Higgs in the MSSM.

In general, in the Standard Model, in addition to the three known left-handed $I = 1/2$ lepton doublets, there could be some number n_s of electroweak-singlet neutrinos. In a compact notation, one can then denote ν_L as the 3-component vector of left-handed $I = 1/2$ neutrinos and χ_R to be the n_s-dimensional vector of electroweak-singlet singlets, taken to be right-handed. In supersymmetric extensions of the Standard Model, the χ_R arise as (conjugates of) the spin 1/2 component fields in (left-handed) gauge-singlet chiral superfields. The general neutrino mass term in the Lagrangian is then given by

$$-\mathcal{L}_m = \frac{1}{2} (\overline{\nu}_L, \overline{\chi}^c_L) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \chi_R \end{pmatrix} + h.c. \tag{2}$$

where M_L is the 3×3 left-handed Majorana-mass matrix, M_R is a $n_s \times n_s$ right-handed Majorana-mass matrix, and M_D is the 3-row by n_s-column Dirac-mass matrix. In general, all of these mass matrices are complex. The anticommutativity of fermion fields and the property that $C\gamma_\mu C^{-1} = -{\gamma_\mu}^T$