the B and D meson masses in the heavy quark effective theory are given in terms of a single nonperturbative parameter $\overline{\Lambda}$,

$$M(B) = m_b + \overline{\Lambda} + \mathcal{O}\left(\frac{\overline{\Lambda}^2}{m_b}\right) ,$$
$$M(D) = m_c + \overline{\Lambda} + \mathcal{O}\left(\frac{\overline{\Lambda}^2}{m_c}\right) ,$$

This allows one to determine the mass difference $m_b - m_c = M(B) - M(D) = 3.4$ GeV up to corrections of order $\overline{\Lambda}^2/m_b - \overline{\Lambda}^2/m_c$. The extraction of the individual quark masses m_b and m_c requires some knowledge of $\overline{\Lambda}$. An estimate of $\overline{\Lambda}$ using QCD sum rules gives $\overline{\Lambda} = 0.57 \pm 0.07$ GeV [10]. The HQET masses with this value of $\overline{\Lambda}$ are $m_b = 4.74 \pm 0.14$ GeV and $m_c = 1.4 \pm 0.2$ GeV, where the spin averaged meson masses $(3M(B^*) + M(B))/4$ and $(3M(D^*) + M(D))/4$ have been used to eliminate the spin-dependent $\mathcal{O}(\overline{\Lambda}^2/m_Q)$ correction terms. The errors reflect the uncertainty in $\overline{\Lambda}$ and the unknown spin-averaged $\mathcal{O}(\overline{\Lambda}^2/m_Q)$ correction. The errors do not include any theoretical uncertainty in the QCD sum rules, which could be large. A quark model estimate suggests that $\overline{\Lambda}$ is the constituent quark mass (≈ 350 MeV), which differs significantly from the sum rule estimate. In HQET, the $1/m_Q$ corrections to heavy meson decay form-factors are also given in terms of $\overline{\Lambda}$. Thus an accurate enough measurement of these form-factors could be used to extract $\overline{\Lambda}$ directly from experiment, which then determines the quark masses up to corrections of order $1/m_Q$.

The quark mass m_Q of HQET can be related to other quark mass parameters using QCD perturbation theory at the scale m_Q. The relation between these masses in QCD perturbation theory at one loop is [11]

$$\begin{aligned} m_Q &= \overline{m}(\mu)\left[1 + \frac{\overline{\alpha}_s(\mu)}{3\pi}\left(3\log \mu^2/m^2 + 4\right)\right] , \\ m_Q &= \widehat{m}(\xi)\left[1 + \frac{\widehat{\alpha}_s(\xi)}{\pi}\frac{\xi+2}{\xi+1}\log(\xi+2)\right], \\ m_Q &= m_P , \end{aligned} \tag{2}$$

where $\overline{\alpha}_s(\mu)$ and $\widehat{\alpha}_s(\xi)$ are the strong interaction coupling constants in the $\overline{\mathrm{MS}}$ and momentum space subtraction schemes, respectively.

D. Light quarks

For light quarks, one can use the techniques of chiral perturbation theory to extract quark mass ratios. The light quark part of the QCD Lagrangian Eq. (1) has a chiral symmetry in the limit that the light quark masses are set to zero, under which left- and right-handed quarks transform independently. The mass term explicitly breaks the chiral symmetry, since it couples the left- and right-handed quarks to each other. A systematic analysis of this explicit chiral symmetry breaking provides some information on the light quark masses.

It is convenient to think of the three light quarks u, d and s as a three component column vector Ψ, and to write the mass term for the light quarks as

$$\overline{\Psi} M \Psi = \overline{\Psi}_L M \Psi_R + \overline{\Psi}_R M \Psi_L, \tag{3}$$

where M is the quark mass matrix M,

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \tag{4}$$

The mass term $\overline{\Psi} M \Psi$ is the only term in the QCD Lagrangian that mixes left- and right-handed quarks. In the limit that $M \to 0$, there is an independent SU(3) flavor symmetry for the left- and right-handed quarks. This $G_\chi = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ chiral symmetry of the QCD Lagrangian is spontaneously broken, which leads to eight massless Goldstone bosons, the π's, K's, and η, in the limit $M \to 0$. The symmetry G_χ is only an approximate symmetry, since it is explicitly broken by the quark mass matrix M. The Goldstone bosons acquire masses which can be computed in a systematic expansion in M in terms of certain unknown nonperturbative parameters of the theory. For example, to first order in M one finds that [12,13]

$$\begin{aligned} m^2_{\pi^0} &= B\,(m_u + m_d) , \\ m^2_{\pi^\pm} &= B\,(m_u + m_d) + \Delta_{em} , \\ m^2_{K^0} = m^2_{\overline{K}^0} &= B\,(m_d + m_s) , \\ m^2_{K^\pm} &= B\,(m_u + m_s) + \Delta_{em} , \\ m^2_\eta &= \frac{1}{3}B\,(m_u + m_d + 4m_s) , \end{aligned} \tag{5}$$

with two unknown parameters B and Δ_{em}, the electromagnetic mass difference. From Eq. (5), one can determine the quark mass ratios [12]

$$\begin{aligned} \frac{m_u}{m_d} &= \frac{2m^2_{\pi^0} - m^2_{\pi^+} + m^2_{K^+} - m^2_{K^0}}{m^2_{K^0} - m^2_{K^+} + m^2_{\pi^+}} = 0.56 , \\ \frac{m_s}{m_d} &= \frac{m^2_{K^0} + m^2_{K^+} - m^2_{\pi^+}}{m^2_{K^0} + m^2_{\pi^+} - m^2_{K^+}} = 20.1 , \end{aligned} \tag{6}$$

to lowest order in chiral perturbation theory. The error on these numbers is the size of the second-order corrections, which are discussed at the end of this section. Chiral perturbation theory cannot determine the overall scale of the quark masses, since it uses only the symmetry properties of M, and any multiple of M has the same G_χ transformation law as M. This can be seen from Eq. (5), where all quark masses occur only in the form Bm, so that B and m cannot be determined separately.

The mass parameters in the QCD Lagrangian have a scale dependence due to radiative corrections, and are renormalization scheme dependent. Since the mass ratios extracted using