chiral perturbation theory use the symmetry transformation property of M under the chiral symmetry G_χ, it is important to use a renormalization scheme for QCD that does not change this transformation law. Any quark mass independent subtraction scheme such as $\overline{\rm MS}$ is suitable. The ratios of quark masses are scale independent in such a scheme.

The absolute normalization of the quark masses can be determined by using methods that go beyond chiral perturbation theory, such as QCD sum rules [3]. Typically, one writes a sum rule for a quantity such as B in terms of a spectral integral over all states with certain quantum numbers. This spectral integral is then evaluated by assuming it is dominated by one (or two) of the lowest resonances, and using the experimentally measured resonance parameters [14]. There are many subtleties involved, which cannot be discussed here [14].

Another method for determining the absolute normalization of the quark masses, is to assume that the strange quark mass is equal to the SU(3) mass splitting in the baryon multiplets [12,14]. There is an uncertainty in this method since in the baryon octet one can use either the Σ–N or the Λ–N mass difference, which differ by about 75 MeV, to estimate the strange quark mass. It is also not possible to relate this definition of mass to the mass parameters in the QCD Lagrangian, which are scale and scheme dependent. It is often simply assumed that the baryon mass splittings give the $\overline{\rm MS}$ quark masses renormalized at a scale $\mu = 1$ GeV.

One can extend the chiral perturbation expansion Eq. (5) to second order in the quark masses M to get a more accurate determination of the quark mass ratios. The meson mass terms in the chiral Lagrangian to second order in the quark mass matrix M are [16]

$$\mathcal{L} = \tfrac{1}{4}{\rm Tr}\left(\chi^\dagger U + \chi U^\dagger\right) + L_6\ \left[{\rm Tr}\left(\chi^\dagger U + \chi U^\dagger\right)\right]^2$$
$$+ L_7\ \left[{\rm Tr}\left(\chi^\dagger U - \chi U^\dagger\right)\right]^2 + L_8\ {\rm Tr}\left(\chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger\right) \quad (7)$$

where $\chi = 2BM$ and U is the exponential of the pion field. Eq. (7) defines the chiral Lagrangian parameters L_{6-8}. The meson masses are obtained by expanding Eq. (7) in a power series in the meson fields.

There is a subtlety that arises at this order [15], because

$$M\left(M^\dagger M\right)^{-1} \det M^\dagger \quad (8)$$

transforms in the same way under G_χ as M, so that one can make the replacement $M \to M(\lambda) = M + \lambda M\left(M^\dagger M\right)^{-1}\det M^\dagger$,

$$M(\lambda) = {\rm diag}\left(m_u(\lambda)\,,\ m_d(\lambda)\,,\ m_s(\lambda)\right)$$
$$= {\rm diag}\left(m_u + \lambda m_d m_s\,,\ m_d + \lambda m_u m_s\,,\ m_s + \lambda m_u m_d\right) \quad (9)$$

One can only determine the ratios $m_i(\lambda)/m_j(\lambda)$ using second-order chiral perturbation theory, not the ratios $m_i/m_j = m_i(\lambda=0)/m_j(\lambda=0)$, since second-order terms in the chiral Lagrangian are indistinguishable from first-order terms with a redefined quark mass. M and $M(\lambda)$ have the same chiral transformation properties, which implies that the meson masses depend on L_{6-8} only in the linear combinations $L_6 - L_7$ and $L_6 + 2L_8$. One linear combination is left undetermined, so one cannot determine the quark mass ratios to second order in chiral perturbation theory without an independent determination of one of the L_i's.

Dimensional analysis can be used to estimate [17] that second-order corrections in chiral perturbation theory due to the strange quark mass are of order $\lambda m_s \sim 0.25$. The ambiguity due to the redefinition Eq. (9) (which corresponds to a second-order correction) can produce a sizeable uncertainty in the ratio m_u/m_d. The lowest-order value $m_u/m_d = 0.56$ gets corrections of order $\lambda m_s(m_d/m_u - m_u/m_d) \sim 30\%$, whereas m_s/m_d gets a smaller correction of order $\lambda m_s(m_u/m_d - m_u m_d/m_s^2) \sim 15\%$. A more quantitative discussion of second-order effects can be found in Refs. 15,16,18. Since the second-order terms have a single parameter ambiguity, the value of m_u/m_d is related to the value of m_s/m_d.

The ratio m_u/m_d is of great interest since there is no strong CP problem if $m_u = 0$. To determine m_u/m_d requires determining the parameters L_{6-8} in the meson Lagrangian Eq. (7), or equivalently, fixing λ in the mass redefinition Eq. (9). There has been considerable effort to determine the chiral Lagrangian parameters accurately enough to determine m_u/m_d, for example from the analysis of the decays $\psi' \to \psi + \pi^0, \eta$, the decay $\eta \to 3\pi$, and by using sum rules [14,19–21]. m_u/m_d can also be extracted from η' decays. This requires the knowledge of chiral lagrangian parameters for the η', such as L_{14} which is defined in [21]. All of these methods have large uncertainties, but indicate that $m_u/m_d \neq 0$. This conclusion has been questioned by Ref. 20 which argues that instanton corrections produce effects like the redefinition Eq. (9), and can explain the experimental observations even if $m_u = 0$.

Eventually, lattice gauge theory methods will be accurate enough to be able to compute meson masses directly from the QCD Lagrangian Eq. (1), and thus determine the light quark masses. For a reliable determination of quark masses, these computations will have to be done with dynamical fermions, and with a small enough lattice spacing that one can accurately compute the relation between lattice and continuum Lagrangians.

The quark masses for light quarks discussed so far are often referred to as current quark masses. Nonrelativistic quark models use constituent quark masses, which are of order 350 MeV for the u and d quarks. Constituent quark masses model the effects of dynamical chiral symmetry breaking, and are not related to the quark mass parameters m_k of the QCD Lagrangian Eq. (1). Constituent masses are only defined in the context of a particular hadronic model.

E. Numerical values and caveats

The quark masses in the particle data listings have been obtained by using the wide variety of theoretical methods outlined