violation in the mixing is described by a single parameter ϵ:

$$| K_L^0 \rangle = \left[(1+\epsilon) \mid K^0 \rangle - (1-\epsilon) \mid \overline{K}^0 \rangle \right] / \left[2(1+|\epsilon|^2) \right]^{1/2} \quad (2a)$$

$$| K_S^0 \rangle = \left[(1+\epsilon) \mid K^0 \rangle + (1-\epsilon) \mid \overline{K}^0 \rangle \right] / \left[2(1+|\epsilon|^2) \right]^{1/2} \quad . \quad (2b)$$

The decay amplitudes are written

$$\langle I=0 \mid T \mid K^0 \rangle = e^{i\delta_0} A_0 \quad (3a)$$

$$\langle I=2 \mid T \mid K^0 \rangle = e^{i\delta_2} A_2 \quad (3b)$$

where δ_I are the $\pi\pi$ scattering phase shifts at the K^0 mass and I is the isospin of the final state. CP violation is measured by (Im A_I/Re A_I). One can then write

$$\eta_{+-} = \epsilon + \epsilon' \quad (4a)$$

$$\eta_{00} = \epsilon - 2\epsilon' \quad (4b)$$

where

$$\epsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{\mathrm{Re}\ A_2}{\mathrm{Re}\ A_0} \left[\frac{\mathrm{Im}\ A_2}{\mathrm{Re}\ A_2} - \frac{\mathrm{Im}\ A_0}{\mathrm{Re}\ A_0} \right] \quad (5)$$

neglecting small corrections of order ϵ' times Re(A_2/A_0). Only two of the three quantities ϵ, (Im A_I/Re A_I) are meaningful because of the ambiguity in defining the phase of K^0. The standard phase convention due to Wu and Yang [3] sets Im $A_0 = 0$. A nonzero value of ϵ' would provide definite evidence for CP violation in the decay amplitudes independent of phase convention.

By applying CPT invariance and unitarity it is possible to relate δ to ϵ and to determine the phases of ϵ. If one assumes the $\Delta S = \Delta Q$ rule (see below "Note on the $\Delta S = \Delta Q$ rule in K^0 Decay") the expression for δ becomes

$$\delta = 2\mathrm{Re}\ \epsilon / (1+ \mid \epsilon \mid^2) \approx 2\mathrm{Re}\ \epsilon \ . \quad (6)$$

This quantity is independent of phase convention and is seen from Eq. (2) to equal $\langle K_L^0 \mid K_S^0 \rangle$. The phase of ϵ is given by

$$\phi(\epsilon) \approx \tan^{-1} \frac{(2\Delta m \tau_s)}{\hbar} = 43.59 \pm 0.15^\circ \quad (7a)$$

while Eq. (5) gives

$$\phi(\epsilon') = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 48 \pm 4^\circ \ . \quad (7b)$$

The approximation in Eq. (7a) depends on the neglect of CP violation in decays other than $K^0 \to 2\pi$ and is known to be good to a few tenths of a degree. Eq. (7a) is evaluated using the values of the $K_L^0 - K_S^0$ mass difference $\Delta m = (0.5333 \pm 0.0027) \times 10^{10} \hbar \mathrm{s}^{-1}$ and the K_S^0 mean life $\tau_s = (0.8926 \pm 0.0012) \times 10^{-10}$s from the current edition. The value of the $\pi\pi$ phase shifts is taken from the fit given by Chell and Olsson [4]. The most important point for the analysis is that $\cos[\phi(\epsilon') - \phi(\epsilon)] \simeq 1$. The consequence of this analysis is that only two real quantities need be measured, the magnitude of ϵ and the value of (ϵ'/ϵ) including its sign. The measured quantity $|\eta_{00}/\eta_{+-}|^2$ which is very close to unity, is given to a good approximation by

$$|\eta_{00}/\eta_{+-}|^2 \approx 1 - 6\mathrm{Re}\ (\epsilon'/\epsilon) = 1 - 6(\epsilon'/\epsilon) \cos\left[\phi(\epsilon') - \phi(\epsilon)\right] \ . \quad (8)$$

Since the cos in Eq. (8) is expected theoretically to be very close to unity it is customary to say that $|\eta_{00}/\eta_{+-}|^2$ determines ϵ'/ϵ.

It is possible to use the values of the ϕ_{+-} and $\phi_{00} - \phi_{+-}$ to set limits on CPT violation. [See Tests of Conservation Laws.]

Models

In the superweak model [5] CP violation is restricted to the mass mixing so that to a high degree of accuracy one expects $\epsilon' = 0$. The phase $\phi(\epsilon)$ is given in this model exactly by Eq. (7a) so that this has sometimes been referred to as the superweak phase; however, as noted above, all CPT invariant models give Eq. (7a) as a very good approximation. In the Standard Model CP violation is entirely due to the phase in the Cabibbo-Kobayashi-Maskawa mixing matrix [6](q.v.). Since CP violation occurs in first order in decay amplitudes and in second order in mass-matrix mixing, one expects a significant nonzero value of ϵ'. The calculation is uncertain partly because m_t and V_{td} are not well known and primarily because of the difficulty of estimating hadronic matrix elements [7]. The theoretical results for ϵ'/ϵ in the standard model are generally in the range 3×10^{-4} to 5×10^{-3}, but may be even lower for large values of m_t.

Fitting procedures

We list measurements of $|\eta_{+-}|$, $|\eta_{00}|$, and $|\eta_{00}/\eta_{+-}|$. Independent information on $|\eta_{+-}|$ and $|\eta_{00}|$ can be obtained from measurements of the K_L^0 and K_S^0 lifetimes (τ) and branching ratios (B) to $\pi\pi$, using the relations

$$|\eta_{+-}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^+\pi^-)}{\tau(K_L^0)} \frac{\tau(K_S^0)}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)} \right]^{1/2} \ , \quad (9a)$$

$$|\eta_{00}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\tau(K_L^0)} \frac{\tau(K_S^0)}{\mathrm{B}(K_S^0 \to \pi^0\pi^0)} \right]^{1/2} \ . \quad (9b)$$

We approximate a global fit to these independent sources by first performing two independent fits: 1) BRFIT, a fit to the K_L^0 branching ratios, rates, and mean life, and 2) ETAFIT, a fit to the $|\eta_{+-}|$, $|\eta_{00}|$, and $|\eta_{+-}/\eta_{00}|$ measurements. The results from fit 1, along with the K_S^0 values from this edition are used to compute values of $|\eta_{+-}|$ and $|\eta_{00}|$ which are included as measurements in the $|\eta_{00}|$ and $|\eta_{+-}|$ sections with a document ID of BRFIT 94. Thus the fit values of $|\eta_{+-}|$ and $|\eta_{00}|$ given in this edition include both the direct measurements and the results from the branching ratio fit.

The process is reversed in order to include the direct $|\eta|$ measurements in the branching ratio fit. The results from