$B \to \overline{D}^{*}\ell^{+}\nu_{\ell}$, so in B decays, form factors have been studied only for $B \to \overline{D}^{*}\ell^{+}\nu_{\ell}$. In this section, we discuss the formalism used in form-factor measurements for the decays $P \to P'\ell\nu_{\ell}$, where P and P' are pseudoscalar mesons, and $P \to V\ell\nu_{\ell}$, where V is a vector meson.

The differential decay rate for $P(Q\overline{q}) \to P'(q'\overline{q})\,\ell\nu_{\ell}$ is

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{q'Q}|^2 k_{P'}^3 |f_+(q^2)|^2}{24\pi^3} \ . \tag{1}$$

Here G_F is the Fermi decay constant, $V_{q'Q}$ is the relevant CKM matrix element, $k_{P'}$ is the momentum of P' in the rest frame of the parent meson, and $f_+(q^2)$ is a vector form factor. (Eq. (1) assumes massless charged leptons, which is almost exact for electrons and a very good approximation for muons, but it is not correct for τ's.) The dominant q^2 dependence comes from the p-wave factor $k_{P'}^3$, which can be written in terms of q^2 and the particle masses. This factor increases the rate at low q^2, which is opposite to the q^2 dependence of the form factor f_+.

The exclusive decay rate for $P \to V\ell\nu_{\ell}$ can be expressed in terms of three q^2-dependent helicity amplitudes, $H_{\pm}(q^2)$ and $H_0(q^2)$, where the subscripts indicate the helicity of either the virtual W or the vector meson. The rate is given by

$$\begin{aligned}
\frac{d\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos_V\, d\chi} &= \frac{3G_F^2 |V_{q'Q}|^2\, k_V\, q^2}{8(4\pi)^4 M^2}\Big\{[(1+\eta\cos\theta_\ell)^2 |H_+(q^2)|^2 \\
&+ (1-\eta\cos\theta_\ell)^2 |H_-(q^2)|^2]\sin^2\theta_V + 4\sin^2\theta_\ell\cos^2\theta_V |H_0(q^2)|^2 - 2\sin^2\theta_\ell\sin^2\theta_V\cos(2\chi)H_+(q^2)H_-(q^2) \\
&- 4\eta\sin\theta_\ell(1+\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi\, H_+(q^2)H_0(q^2) + 4\eta\sin\theta_\ell(1-\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi\, H_-(q^2)H_0(q^2)\Big\} \ .
\end{aligned} \tag{2}$$

Here M is the mass of the parent meson, k_V is the momentum of the vector meson and is a function of q^2, and the factor η=+1 ($\eta = -1$) applies to B (D) decays. The angles θ_ℓ, θ_V, and χ are defined in Fig. 1. The helicity amplitudes $H_{\pm}$ and H_0 can be expressed in terms of two axial-vector form factors, $A_1(q^2)$ and $A_2(q^2)$, and a vector form factor $V(q^2)$:

$$\begin{aligned}
H_{\pm}(q^2) =& (M+m)A_1(q^2) \mp \frac{2M\, k_V}{(M+m)} V(q^2) \\
H_0(q^2) =& \frac{1}{2m\sqrt{q^2}}\Big[(M^2 - m^2 - q^2)(M+m)A_1(q^2) - \frac{4M^2\, k_V^2}{(M+m)} A_2(q^2)\Big] \ ,
\end{aligned} \tag{3}$$

where m is the mass of the daughter meson. The form factors f_+, A_1, A_2, and V are dimensionless.

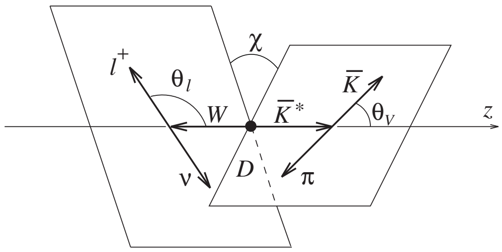

Figure 1: Definition of the angles θ_ℓ, θ_V, and χ. The decay $D \to \overline{K}^{*}\ell^{+}\nu_{\ell}$ is used as an example. The polar angles θ_ℓ and θ_V are defined in the rest frames of the virtual W and the $\overline{K}^{*}$, respectively, and χ is the azimuthal angle between the projections of the lepton and the kaon momentum vectors in the plane perpendicular to z.

The $V-A$ coupling results in a larger amplitude to produce a negative-helicity vector meson in $c\overline{q}$ or $b\overline{q}$ decay than one of positive helicity: $|H_-| > |H_+|$. This difference produces a forward-backward asymmetry for the charged lepton in the virtual-W rest frame, since the net angular momentum along the decay axis of the initial heavy meson must be zero. For $D(c\overline{q})$ decays, a positively charged (right-handed) lepton is produced in association with a left-handed daughter s or d quark, resulting in a softer energy spectrum for the charged lepton than for the neutrino after boosting the lepton energy into the D rest frame. (A similar argument shows that the shape of the spectrum is the same for a $\overline{D}$ decay.) For $\overline{B}(b\overline{q})$ decays, a negatively charged (left-handed) lepton is produced in association with a left-handed daughter quark, giving a harder energy spectrum for the charged lepton than for the neutrino in the B rest frame. In $P \to P'\ell\nu_{\ell}$ decays, there is no asymmetry, since the P' meson can only have helicity zero. Thus, the effect of $V-A$ is to soften the inclusive lepton spectrum in D decays and to harden it in B decays. It is useful to define rates for decays into specific helicity states:

$$\Gamma_i = \frac{G_F^2 |V_{q'Q}|^2}{96\pi^3} \int dq^2\, k_V \frac{q^2}{M^2} |H_i(q^2)|^2 \ . \tag{4}$$