The values of $\mathcal{B}_{SL}$ given in Table 1 and Table 2 are lower than theoretical predictions, which give [15] $\mathcal{B}_{SL} \geq 12.5\%$. Such calculations, however, are difficult partly because they must determine the total hadronic rate, which has uncertainties associated with both perturbative and nonperturbative QCD effects. The source of the discrepancy between theory and experiment is not yet understood.

The semileptonic branching fraction can be used to calculate $|V_{cb}|$. The fraction is converted into a decay rate using the measured B lifetime, and the rate is then compared with model predictions, which depend on $|V_{cb}|^2$. Whereas the model dependence in the determination of $\mathcal{B}_{SL}$ is associated with the predicted shapes of momentum spectra, the extraction of $|V_{cb}|$ is also sensitive to the uncertainty in the normalization of the predicted rate. It is difficult to assign errors to rate predictions based on quark-model calculations. Quite often, a nominal theoretical error of 20% in the rate is assumed, leading to a 10% theoretical error on $|V_{cb}|$. This uncertainty is separate from the model dependence, if any, in extracting the branching fraction. The CLEO-II preliminary results (assuming the value [16] $\tau_B = 1.49 \pm 0.04$ ps which is used in Sec. 28 "The Cabibbo-Kobayashi-Maskawa Mixing Matrix") for $B \to X_{\overline{c}}\ell^+\nu_\ell$ in Table 1 give [17]

$$|V_{cb}| = (0.042 \pm 0.001 \pm 0.004)\sqrt{\frac{1.49}{\tau_B(\text{ps})}} \qquad (1)$$

for both ACCMM and ISGW**. (The value for V_{cb} in Sec. 28 "The Cabibbo-Kobayashi-Maskawa Mixing Matrix" is not based on this measurement but on the $B \to \overline{D}^*\ell^+\nu_\ell$ rate at zero D^* recoil velocity.)

Currently, there is interest in using the methods of HQET to obtain a value of $|V_{cb}|$ from the inclusive spectrum with a better understood error. Luke and Savage [18] use the inclusive branching fraction to obtain the range $0.038 < |V_{cb}| < 0.054$, where we have corrected their result to $\tau_B = 1.49$ ps. Shifman, Uraltsev, and Vainshtein [19] argue that the present theoretical uncertainty on $|V_{cb}|$ may be as low as 5% and can be made even lower. We discuss the determination of $|V_{cb}|$ using exclusive processes in Section IV.4.

Table 3 lists additional inclusive B branching fractions that involve leptons. The decay $X_{\overline{b}} \to X\tau^+\nu_\tau$ has been difficult to observe, due to the presence of two neutrinos. However, ALEPH has used decay configurations with a large missing energy in the b-hadron rest frame. In Z decays, the boost of the b hadron to the lab frame can result in a very large missing energy (10–30 GeV) for the hemisphere containing the decay. By looking for events with both a large missing energy and a tagged b hadron (using vertex detector information), ALEPH has been able to measure $\mathrm{B}(X_{\overline{b}} \to X\tau^+\nu_\tau)$ with a value consistent with standard model expectations. (The preliminary ALEPH result in Table 3 includes data from the original measurement, so the two results cannot be averaged.)

The lepton endpoint region and the determination of $|V_{ub}|$

The determination of $|V_{ub}|$ is one of the most important and challenging measurements in B physics. For $|V_{ub}/V_{cb}| \approx 0.1$, the rate for $B \to X_{\overline{u}}\ell^+\nu_\ell$, where $X_{\overline{u}}$ is a charmless hadronic system, is expected to be only about 1% of the inclusive semileptonic rate. By working in the region at and beyond the lepton-momentum-spectrum endpoint for $B \to X_{\overline{c}}\ell^+\nu_\ell$ processes, however, one gains enormously in sensitivity to $B \to X_{\overline{u}}\ell^+\nu_\ell$ decays.

Although the advantages of working in this endpoint region ($2.3 < p_\ell < 2.6$ GeV/c) are decisive, there are also disadvantages. A major difficulty is the need to convert the measured rate for this tiny portion of phase space into a value of $|V_{ub}|$. This calculation can be performed using either inclusive or exclusive models, but both have substantial uncertainties in predicting the rate in the endpoint region. Inclusive models are expected to be fairly reliable, if one considers a large enough part of phase space, but they may not be reliable in the endpoint region, which some theorists argue [13] is dominated by a small set of exclusive channels ($B \to \rho\ell^+\nu_\ell$, $B^+ \to \omega\ell^+\nu_\ell$, and $B \to \pi\ell^+\nu_\ell$). Alternatively, exclusive models can be used to predict the sum of contributions of individual modes in this region. However, large uncertainties exist in the calculations of the rates for exclusive modes, and some of the observed rate may be due to nonresonant final states [24]. The exclusive calculations here are more difficult than those for $B \to X_{\overline{c}}\ell^+\nu_\ell$, because the range of recoil velocities available to the light final-state mesons in a $B \to X_{\overline{u}}\ell^+\nu_\ell$ transition is much larger than for the charm mesons in a $B \to X_{\overline{c}}\ell^+\nu_\ell$ decay. One therefore expects a much larger variation in the (poorly known) form factors that enter into the decay rate. As a result, measurements of $|V_{ub}|$ are currently quite model dependent, and there is substantial variation among values obtained using different models.

The analysis of the endpoint region, although an "inclusive" measurement, is quite different from the measurement of $\mathcal{B}_{SL}$ described in the previous section: at the $\Upsilon(4S)$, nonresonant (continuum) processes produce high-momentum leptons that constitute an enormous background (relative to a $B \to X_{\overline{u}}\ell^+\nu_\ell$ signal) unless suppressed by kinematic cuts. The signal efficiency of these cuts is model dependent, unlike the very loose cuts used in the analysis of the inclusive lepton spectrum. In particular, the efficiency depends on the q^2 distribution of the signal events, so the value obtained for the rate in the endpoint region depends on the shape assumed for this distribution. The most important sensitivity to models, however, arises when one converts the rate to $|V_{ub}|$.