the shower develops. They are often represented as the sum of two Gaussians, and Grindhammer [37] describes them with the function

$$f(r) = \frac{2r\,R^2}{(r^2 + R^2)^2} \ , \tag{10.26}$$

where R is a phenomenological function of x/X_0 and $\ln E$.

10.9. Muon energy loss at high energy

At sufficiently high energies, radiative processes become more important than ionization for all charged particles. For muons and pions in materials such as iron, this "critical energy" occurs at several hundred GeV. Radiative effects dominate the energy loss of energetic muons found in cosmic rays or produced at the newest accelerators. These processes are characterized by small cross sections, hard spectra, large energy fluctuations, and the associated generation of electromagnetic and (in the case of photonuclear interactions) hadronic showers. As a consequence, at these energies the treatment of energy loss as a uniform and continuous process is for many purposes inadequate.

It is convenient to write the average rate of muon energy loss as [38]

$$-dE/dx = a(E) + b(E)\,E \ . \tag{10.27}$$

Here $a(E)$ is the ionization energy loss given by Eq. (10.1), and $b(E)$ is the sum of e^+e^- pair production, bremsstrahlung, and photonuclear contributions. To the approximation that these slowly-varying functions are constant, the mean range x_0 of a muon with initial energy E_0 is given by

$$x_0 \approx (1/b)\ln(a + bE_0) \ . \tag{10.28}$$

Figure 10.10 shows contributions to $b(E)$ for iron. Since $a(E) \approx 0.002$ GeV g^{-1} cm^2, $b(E)E$ dominates the energy loss above several hundred GeV, where $b(E)$ is nearly constant. The rate of energy loss for muons in hydrogen, uranium, and iron is shown in Fig. 10.11 [39].

QED calculations of cross sections for bremsstrahlung and e^+e^- pair production have long been known, but were much improved around 1970 to meet the needs of cosmic ray physics [40–44]. Rozental showed that the screened atomic electron contribution could be included by replacing Z^2 with $Z(Z+1.2)$ in the nuclear bremsstrahlung cross sections and by $Z(Z+1.3)$ in the case of e^+e^- pair production [45], and that other corrections might reduce the cross section by as much as 5%. We take this as the present uncertainty. Cross sections for both processes have been evaluated independently by Tsai [27].

A comparison of various improvements to the Bethe-Heitler formula is given by Wright [46]. For muon energies above 100 GeV, $\mu^+\mu^-$ pair production is also possible. This process is potentially troublesome