Between $\mu - \sigma$ and $\mu + \sigma$ lies 68.27% of the probability. The half-width at half-maximum is $\sqrt{2 \ln 2}\, \sigma = 1.18\sigma$.

For n Gaussian random variables x_i, the joint p.d.f. is the *multivariate Gaussian*:

$$f(\boldsymbol{x}; \boldsymbol{\mu}, V) = \frac{1}{(2\pi)^{n/2}}\ |V|^{-1/2} \tag{16.20}$$

$$\times\ \exp\left[-\tfrac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^T\ V^{-1}\ (\boldsymbol{x} - \boldsymbol{\mu})\right]\ ,\ |V| \neq 0\ .$$

V is the $n \times n$ *covariance matrix*; $V_{ij} \equiv E[(x_i - \mu_i)(x_j - \mu_j)] \equiv \rho_{ij}\ \sigma_i\, \sigma_j$, and $V_{ii} = \mathrm{Var}(x_i)$; $|V|$ is the determinant of V. For $n = 2$, this becomes

$$f(x_1, x_2;\ \mu_1, \mu_2, \sigma_1, \sigma_2, \rho) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}$$

$$\times\ \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(x_1-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2}\right]\right\}\ . \tag{16.21}$$

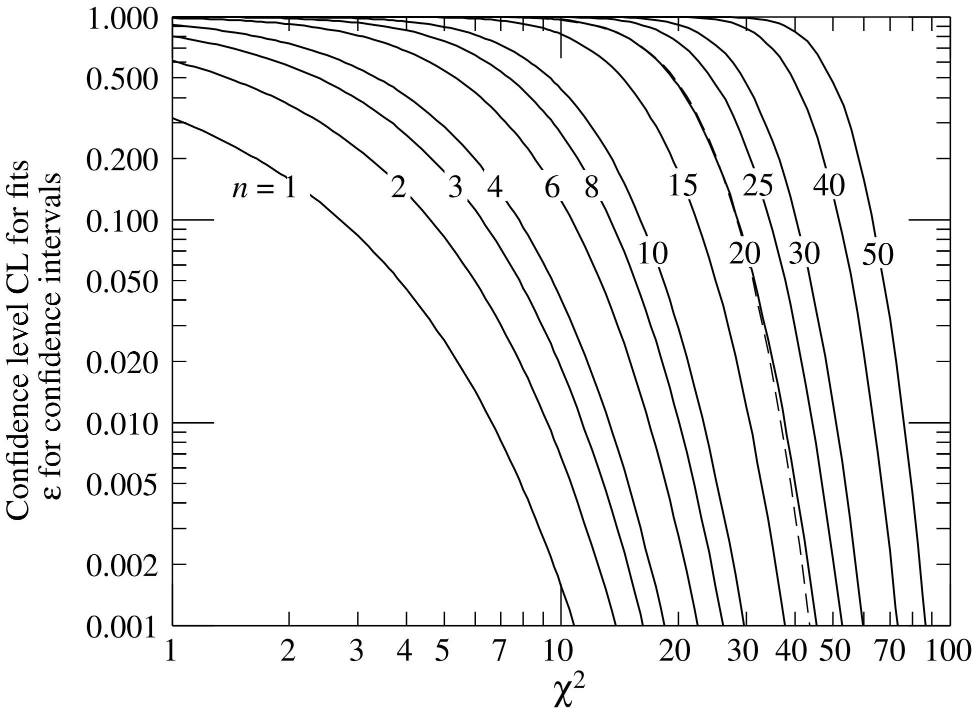

Figure 16.1: The confidence level versus χ^2 for n degrees of freedom, as defined in Eq. (16.22). The curve for a given n gives the probability that a value at least as large as χ^2 will be obtained in an experiment; *e.g.*, for $n = 10$, a value $\chi^2 \gtrsim 18$ will occur in 5% of a large number of experiments.