where

$$(S_{11},\, S_{12},\, S_{22}) = \sum (1,\, x_i,\, x_i^2)/\sigma_i^2 \ , \tag{17.22a}$$

$$(g_1,\, g_2) = \sum (1,\, x_i) y_i/\sigma_i^2 \ . \tag{17.22b}$$

and

$$D = S_{11}\, S_{22} - S_{12}^2 \ , \tag{17.23}$$

and the covariance matrix of the fitted parameters is:

$$\begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix} = \frac{1}{D} \begin{pmatrix} S_{22} & -S_{12} \\ -S_{12} & S_{11} \end{pmatrix} \ . \tag{17.24}$$

17.5. Errors and confidence intervals

One measures a physical quantity under the assumption that a "true answer" $\alpha_{\rm actual}$ exists. The experimental result $\widehat{\alpha}_{\rm exp}$ samples a distribution with p.d.f. $f(\widehat{\alpha}; \alpha)$, where the physical quantity appears as the parameter α. As shown in Fig. 17.1, for every value of the parameter α we can find two numbers $\gamma_1(\alpha, \varepsilon)$ and $\gamma_2(\alpha, \varepsilon)$ such that repeated experiments would produce results in the interval $\gamma_1 < \widehat{\alpha} < \gamma_2$ a fraction $1 - \varepsilon$ of the time, where

$$1 - \varepsilon = \int_{\gamma_1}^{\gamma_2} f(\widehat{\alpha}; \alpha)\, d\widehat{\alpha} \ . \tag{17.28}$$

This is equivalent to the statement that the *confidence interval* $c_1 < \alpha < c_2$ includes $\alpha_{\rm actual}$ with probability $1 - \varepsilon$ [1,6]. In this approach α is a parameter, not a statistical variable. Instead, c_1 and c_2 vary from experiment to experiment and *are* statistical variables.

The choice of γ_1 and γ_2 can be made in an infinite number of ways, but for a given distribution there are usually other criteria. For example, for a Gaussian distribution the choice of symmetric limits minimizes the interval.

Confidence intervals for common distributions are easily available from mathematical libraries (IMSL, NAG, CERNLIB) and personal computer spreadsheets such as Microsoft® Excel [12].

17.5.1. *Gaussian errors*:

For Gaussian errors, we can define a *confidence interval* for a parameter α as the shortest interval which has probability *at least* $1 - \varepsilon$ (ε is called the confidence coefficient) of covering the true value of α. For estimates of a single variable μ with known σ

$$1 - \varepsilon = \int_{\widehat{\mu}-\delta}^{\widehat{\mu}+\delta} f(x;\, \widehat{\mu},\, \sigma^2)\, dx = \mathrm{erf}\left(\frac{x - \widehat{\mu}}{\sqrt{2}\,\sigma}\right) \tag{17.29}$$