23. KINEMATICS

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Throughout this section units are used in which $\hbar = c = 1$. The following conversions are useful: $\hbar c = 197.3$ MeV fermi, $(\hbar c)^2 = 0.3894\ (\mathrm{GeV})^2$ mb.

23.1. Lorentz transformations

The energy E and 3-momentum $\boldsymbol{p}$ of a particle of mass m form a 4-vector $p = (E, \boldsymbol{p})$ whose square $p^2 \equiv E^2 - |\boldsymbol{p}|^2 = m^2$. The velocity of the particle is $\boldsymbol{\beta} = \boldsymbol{p}/E$. The energy and momentum $(E^*, \boldsymbol{p}^*)$ viewed from a frame moving with velocity $\boldsymbol{\beta}_f$ are given by

$$\begin{pmatrix} E^* \\ p_\parallel^* \end{pmatrix} = \begin{pmatrix} \gamma_f & -\gamma_f\beta_f \\ -\gamma_f\beta_f & \gamma_f \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix} , \quad p_T^* = p_T , \tag{23.1}$$

where $\gamma_f = (1-\beta_f^2)^{-1/2}$ and p_T $(p_\parallel)$ are the components of $\boldsymbol{p}$ perpendicular (parallel) to $\boldsymbol{\beta}_f$. The scalar product of two 4-vectors $p_1 \cdot p_2 = E_1E_2 - \boldsymbol{p}_1 \cdot \boldsymbol{p}_2$ is invariant (frame independent).

In the collision of two particles of masses m_1 and m_2 the total center-of-mass energy can be expressed in the Lorentz-invariant form

$$\begin{aligned} E_{\mathrm{cm}} &= \left[(E_1+E_2)^2 - (\boldsymbol{p}_1+\boldsymbol{p}_2)^2\right]^{1/2} , \\ &= \left[m_1^2 + m_2^2 + 2E_1E_2(1-\beta_1\beta_2\cos\theta)\right]^{1/2} , \end{aligned} \tag{23.2}$$

where θ is the angle between the particles. In the frame where one particle (of mass m_2) is at rest (lab frame),

$$E_{\mathrm{cm}} = (m_1^2 + m_2^2 + 2E_{1\,\mathrm{lab}}\, m_2)^{1/2} . \tag{23.3}$$

The velocity in the lab of the center-of-mass frame is

$$\boldsymbol{\beta}_{\mathrm{cm}} = \boldsymbol{p}_{\mathrm{lab}}/(E_{1\,\mathrm{lab}} + m_2) , \tag{23.4}$$

and

$$\gamma_{\mathrm{cm}} = (E_{1\,\mathrm{lab}} + m_2)/E_{\mathrm{cm}} . \tag{23.5}$$

23.2. Center-of-mass energy and momentum

A beam of particles with mass m and momentum $p_{\,\mathrm{beam}}$ is incident on a fixed target consisting of particles with mass M. The energy of the beam particles $E_{\,\mathrm{beam}}$, the total center-of-mass energy E_{cm}, and center of mass momentum of one of the particles p_{cm} are given by

$$E_{\,\mathrm{beam}} = \sqrt{p_{\,\mathrm{beam}}^2 + m^2} \tag{23.6}$$

$$E_{\mathrm{cm}} = \sqrt{m^2 + 2E_{\,\mathrm{beam}}M + M^2} \tag{23.7}$$