23.5.1. *Two-body reactions*:

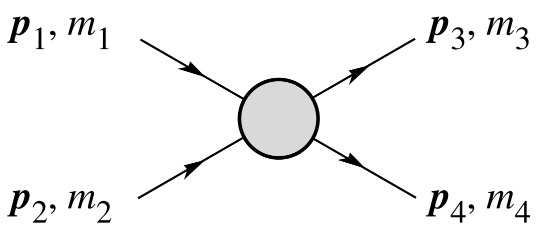

Figure 23.5: Variable definitions for a two-body final state.

Two particles of momenta p_1 and p_2 and masses m_1 and m_2 scatter to particles of momenta p_3 and p_4 and masses m_3 and m_4; the Lorentz-invariant Mandelstam variables are defined by

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\ &= m_1^2 + 2E_1E_2 - 2\boldsymbol{p}_1 \cdot \boldsymbol{p}_2 + m_2^2 \ , \end{aligned} \tag{23.28}$$

$$\begin{aligned} t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\ &= m_1^2 - 2E_1E_3 + 2\boldsymbol{p}_1 \cdot \boldsymbol{p}_3 + m_3^2 \ , \end{aligned} \tag{23.29}$$

$$\begin{aligned} u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 \\ &= m_1^2 - 2E_1E_4 + 2\boldsymbol{p}_1 \cdot \boldsymbol{p}_4 + m_4^2 \ , \end{aligned} \tag{23.30}$$

and they satisfy

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 \ . \tag{23.31}$$

The two-body cross section may be written as

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \, \frac{1}{|\boldsymbol{p}_{1\mathrm{cm}}|^2} \, |\mathscr{M}|^2 \ . \tag{23.32}$$

In the center-of-mass frame

$$\begin{aligned} t &= (E_{1\mathrm{cm}} - E_{3\mathrm{cm}})^2 - (p_{1\mathrm{cm}} - p_{3\mathrm{cm}})^2 \\ &\qquad - 4p_{1\mathrm{cm}}\, p_{3\mathrm{cm}} \ \sin^2(\theta_{\mathrm{cm}}/2) \\ &= t_0 - 4p_{1\mathrm{cm}}\, p_{3\mathrm{cm}} \ \sin^2(\theta_{\mathrm{cm}}/2) \ , \end{aligned} \tag{23.33}$$

where θ_{cm} is the angle between particle 1 and 3. The limiting values t_0 ($\theta_{\mathrm{cm}} = 0$) and t_1 ($\theta_{\mathrm{cm}} = \pi$) for $2 \to 2$ scattering are