The second form is obtained using the identity $dy/dp_z = 1/E$, and the third form represents the average over ϕ.

Feynman's x variable is given by

$$x = \frac{p_z}{p_{z\,\max}} \approx \frac{E + p_z}{(E + p_z)_{\max}} \; ; \tag{23.41}$$

in the center-of-mass frame,

$$x \approx \frac{2p_{z\,\mathrm{cm}}}{\sqrt{s}} \approx \frac{2m_T \sinh y_{\mathrm{cm}}}{\sqrt{s}} \; . \tag{23.42}$$

For y_{cm} such that $e^{-2y_{\mathrm{cm}}} \ll 1$,

$$x \approx \frac{m_T}{\sqrt{s}} e^{y_{\mathrm{cm}}} \tag{23.43}$$

and

$$(y_{\mathrm{cm}})_{\max} = \ln\left(\sqrt{s}/m\right) \; . \tag{23.44}$$

The definition of rapidity [Eq. (23.39)] may be expanded to obtain

$$\begin{aligned} y &= \frac{1}{2} \ln \frac{\cos^2(\theta/2) + m^2/4p^2 + \ldots}{\sin^2(\theta/2) + m^2/4p^2 + \ldots} \\ &\approx -\ln \tan(\theta/2) \equiv \eta \end{aligned} \tag{23.45}$$

if the particle has zenith angle θ. The pseudorapidity η defined by the second line is approximately equal to the rapidity y for $p \gg m$ and $\theta \gg 1/\gamma$, and in any case can be measured when the mass and momentum of the particle is unknown. From the definition one can obtain the identities

$$\sinh\eta = \cot\theta \tag{23.46}$$

$$\cosh\eta = 1/\sin\theta \tag{23.47}$$

$$\tanh\eta = \cos\theta \; . \tag{23.48}$$

23.5.3. ***Partial waves***: The amplitude in the center of mass for elastic scattering of spinless particles may be expanded in Legendre polynomials

$$f(k,\theta) = \frac{1}{k} \sum_{\ell} (2\ell + 1) a_\ell P_\ell(\cos\theta) \; , \tag{23.49}$$

where k is the c.m. momentum, θ is the c.m. scattering angle, $a_\ell = (\eta_\ell e^{2i\delta_\ell} - 1)/2i$, $0 \le \eta_\ell \le 1$, and δ_ℓ is the phase shift of the ℓ^{th} partial wave. For purely elastic scattering, $\eta_\ell = 1$. The differential cross section is

$$\frac{d\sigma}{d\Omega} = |f(k,\theta)|^2 \; . \tag{23.50}$$