24.2. e^+e^- annihilation

For pointlike spin-1/2 fermions in the c.m., the differential cross section for $e^+e^- \to f\overline{f}$ via single photon annihilation is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\,\beta\,\left[1+\cos^2\theta+(1-\beta^2)\sin^2\theta\right]Q_f^2\ , \tag{24.11}$$

where β is the velocity of the final state fermion in the center of mass and Q_f is the charge of the fermion in units of the proton charge. For $\beta \to 1$,

$$\sigma = \frac{4\pi\alpha^2}{3s}\,Q_f^2 = \frac{86.8\,Q_f^2\ nb}{s(\mathrm{GeV}^2)}\ . \tag{24.12}$$

At higher energies the Z^0 (mass M_Z and width Γ_Z) must be included, and the differential cross section for $e^+e^- \to f\overline{f}$ becomes

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\,\beta\,\Bigg[&Q_f^2[1+\cos^2\theta+(1-\beta^2)\,\sin^2\theta\,]\\ &-2Q_f\,\chi_1\Big\{VV_f\,[1+\cos^2\theta+(1-\beta^2)\,\sin^2\theta\,]-2a_f\,\beta\cos\theta\Big\}\\ &+\chi_2\Big\{V_f^2\,(1+V^2)\,[1+\cos^2\theta+(1-\beta^2)\,\sin^2\theta\,]\\ &+\beta^2\,a_f^2(1+V^2)\,[1+\cos^2\theta\,]-8\beta\,VV_f\,a_f\cos\theta\Big\}\Bigg]\ ,\end{aligned} \tag{24.13}$$

$$\chi_1 = \frac{1}{16\sin^2\theta_W\cos^2\theta_W}\ \frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2}\ , \tag{24.14}$$

$$\chi_2 = \frac{1}{256\sin^4\theta_W\cos^4\theta_W}\ \frac{s^2}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2}\ , \tag{24.15}$$

$$V = -1+4\sin^2\theta_W\ , \tag{24.16}$$

$$a_f = 2T_{3f}\ , \tag{24.17}$$

$$V_f = 2T_{3f}-4Q_f\sin^2\theta_W\ , \tag{24.18}$$

where the subscript f refers to the particular fermion and

$$T_3 = +1/2 \quad \text{for } \nu_e,\ \nu_\mu,\ \nu_\tau,\ u,\ c,\ t, \tag{24.19a}$$

$$T_3 = -1/2 \quad \text{for } e^-,\ \mu^-,\ \tau^-,\ d,\ s,\ b. \tag{24.19b}$$