24.3. Two-photon process at e^+e^- colliders

When an e^+ and an e^- collide with energies E_1 and E_2, they emit dn_1 and dn_2 virtual photons with energies ω_1 and ω_2 and 4-momenta q_1 and q_2. In the equivalent photon approximation, the cross section for $e^+e^- \to e^+e^-X$ is related to the cross section for $\gamma\gamma \to X$ by (Ref. 1)

$$d\sigma_{e^+e^-\to e^+e^-X}(s) = dn_1\, dn_2\, d\sigma_{\gamma\gamma\to X}(W^2) \tag{24.20}$$

where $s = 4E_1E_2$, $W^2 = 4\omega_1\omega_2$ and

$$dn_i = \frac{\alpha}{\pi}\left[1 - \frac{\omega_i}{E_i} + \frac{\omega_i^2}{2E_i^2} - \frac{m_e^2\omega_i^2}{(-q_i^2)E_i^2}\right]\frac{d\omega_i}{\omega_i}\,\frac{d(-q_i^2)}{(-q_i^2)}\ . \tag{24.21}$$

After integration (including that over q_i^2 in the region $m_e^2\omega_i^2/E_i(E_i-\omega_i) \le -q_i^2 \le (-q^2)_{\rm max}$), the cross section is

$$\begin{aligned}
\sigma_{e^+e^-\to e^+e^-X}(s) = &\frac{\alpha^2}{\pi^2}\int_{z_{th}}^1 \frac{dz}{z}\left[f(z)\left(\ln\frac{(-q^2)_{\rm max}}{m_e^2 z} - 1\right)^2\right.\\
&\left. - \tfrac{1}{3}\left(\ln\frac{1}{z}\right)^3\right]\sigma_{\gamma\gamma\to X}(zs)\ ;\\
f(z) = &\left(1 + \tfrac{1}{2}z\right)^2 \ln\frac{1}{z} - \tfrac{1}{2}(1-z)(3+z)\ ;\\
z = &\frac{W^2}{s}\ .
\end{aligned} \tag{24.22}$$

The quantity $(-q^2)_{\rm max}$ depends on properties of the produced system X, in particular, $(-q^2)_{\rm max} \sim m_\rho^2$ for hadron production ($X = h$) and $(-q^2)_{\rm max} \sim W^2$ for lepton pair production ($X = \ell^+\ell^-$, $\ell = e,\ \mu,\ \tau$).

For production of a resonance of mass m_R and spin $J \ne 1$

$$\begin{aligned}
\sigma_{e^+e^-\to e^+e^-R}(s) = &(2J+1)\frac{8\alpha^2\Gamma_{R\to\gamma\gamma}}{m_R^3}\\
&\times\left[f(m_R^2/s)\left(\ln\frac{s m_V^2}{m_e^2 m_R^2} - 1\right)^2 - \frac{1}{3}\left(\ln\frac{s}{m_R^2}\right)^3\right]
\end{aligned} \tag{24.23}$$

where m_V is the mass that enters into the form factor of the $\gamma\gamma \to R$ transition: $m_V \sim m_\rho$ for $R = \pi^0,\ \rho^0,\ \omega,\ \phi,\ \ldots$, $m_V \sim m_R$ for $R = c\bar{c}$ or $b\bar{b}$ resonances.