α_s at the heavy quark mass scale. The most precise data come from the decay widths of the 1^{--} $J/\psi(1S)$ and Υ resonances. Data are available for the Υ, Υ', Υ'' and ψ. The result is very sensitive to α_s and the data are sufficiently precise ($R_\mu(\Upsilon) = 32.5 \pm 0.9$) [44] that the theoretical errors will dominate. There are theoretical corrections to this simple formula due to the relativistic nature of the $Q\overline{Q}$ system; $v^2/c^2 \sim 0.1$ for the Υ. They are more severe for the ψ. There are also nonperturbative corrections of the form Λ^2/m_Υ^2; again these are more severe for the ψ. A fit to Υ, Υ', and Υ'' [45] gives $\alpha_s(M_Z) = 0.108 \pm 0.001(\text{expt.})$. The results from each state separately and also from the ψ are consistent with each other. There is an uncertainty of order ± 0.005 from the choice of scale; the error from v^2/c^2 corrections is a little larger. $\alpha_s(M_Z) = 0.108 \pm 0.010$ is a fair representation of the total error including the possibility of nonperturbative corrections.

25.7. Perturbative QCD in e^+e^- collisions

The total cross section for $e^+e^- \to$ hadrons is obtained (at low values of $\sqrt{s}$) by multiplying the muon-pair cross section by the factor $R = 3\Sigma_q e_q^2$. The higher-order QCD corrections to this quantity have been calculated, and the results can be expressed in terms of the factor:

$$R = R^{(0)} \left[1 + \frac{\alpha_s}{\pi} + C_2 \left(\frac{\alpha_s}{\pi}\right)^2 + C_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \cdots \right] , \qquad (25.15)$$

where $C_2 = 1.411$ and $C_3 = -12.8$ [46].

$R^{(0)}$ can be obtained from the formula for $d\sigma/d\Omega$ for $e^+e^- \to f\overline{f}$ by integrating over Ω. The formula is given in Sec. 24.2 of this *Review*. This result is strictly only correct in the zero-quark-mass limit. The $\mathcal{O}(\alpha_s)$ corrections are also known for massive quarks [47].

A comparison of the theoretical prediction of Eq. (25.15) (corrected for the b-quark mass) with all the available data at values of $\sqrt{s}$ between 20 and 65 GeV gives [48] $\alpha_s(35\text{ GeV}) = 0.146 \pm 0.030$. The principal advantage of determining α_s from R in e^+e^- annihilation is that there is no dependence on fragmentation models, jet algorithms, *etc.*

Measurements of the ratio of hadronic to leptonic width of the Z at LEP Γ_h/Γ_μ probe the same quantity as R. Using the average of $\Gamma_h/\Gamma_\mu = 20.781 \pm 0.049$ gives $\alpha_s(M_Z) = 0.124 \pm 0.007$ [49]. There are theoretical errors arising from the values of the top quark and Higgs masses which enter due to electroweak corrections to the Z width and from the choice of scale. The dominant error is the first of these (± 0.005).

An alternative method of determining α_s in e^+e^- annihilation is from measuring quantities that are sensitive to the relative rates of