two-, three-, and four-jet events. In addition to simply counting jets, there are many possible choices of such "shape variables": thrust [50], energy-energy correlations [51], planar triple-energy correlations [52], average jet mass, *etc.* All of these are infrared safe, which means they can be reliably calculated in perturbation theory. The starting point for all these quantities is the multijet cross section. For example, at order α_s, for the process $e^+e^- \to qqg$:

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} , \tag{25.16}$$

where

$$x_i = \frac{2E_i}{\sqrt{s}} \tag{25.17}$$

are the center-of-mass energy fractions of the final-state (massless) quarks. A distribution in a "three-jet" variable, such as those listed above, is obtained by integrating this differential cross section over an appropriate phase space region for a fixed value of the variable.

There are many methods used by the LEP groups [53–56] to determine α_s from the event topology. The jet-counting algorithm originally introduced by the JADE collaboration [57] has been used by the LEP groups.

The scale M at which $\alpha_s(M)$ is to be evaluated is not clear. The invariant mass of a typical jet (or $\sqrt{sy_0}$) is probably a more appropriate choice than the e^+e^- center of mass energy. If the value is allowed to float in the fit to the data, the data tend to prefer values of order $\sqrt{s}/10$ [63]; the exact value depends on the variable that is fitted. The dominant uncertainties arise from the choice of M and from the freedom in the fragmentation Monte Carlos. An average of many variables and all the LEP experiments [65] gives $\alpha_s(M_Z) = 0.119 \pm 0.006$. The dominant error is theoretical. The situation might improve if complete perturbative calculations to order α_s^3 became available.

The perturbative QCD formulae can break down in special kinematical configurations. For example, the thrust distribution contains terms of the type $\alpha_s \ln^2(1-T)$. The higher orders in the perturbation expansion contains terms of order $\alpha_s^n \ln^m(1-T)$. For $T \sim 1$ (the region populated by 2-jet events) the perturbation expansion is unreliable. The terms with $n \leq m$ can be summed to all orders in α_s [66]. The resummed results give better agreement with the data at large values of T [67]. Such resummed results are not available for all of the shape variables [69], but fits using those available yields a LEP average of $\alpha_s(M_Z) = 0.124 \pm 0.006$ [65]. We will use the average of $\alpha_s(M_Z) = 0.121 \pm 0.006$. Since the error is dominantly theoretical, it does not reduce in the average.