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On the current value of the Fermi coupling constant

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Proposal to correct the PDG
and NIST/CODATA tables

1. In the last edition of the Review of Particle Physics [1] there are two different values of the Fermi coupling constant. The first estimate presented in the Table 1.1 (page 77) reads as

$$G_F/(\hbar c)^3 = 1.16639(1) \times 10^{-5}\,\mathrm{GeV}^{-2}, \tag{1}$$

and the other value

$$G_F/(\hbar c)^3 = 1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2} \tag{2}$$

is presented there in the Sec. 10 "Electroweak model and constraints on new physics" (page 98).

From the caption of the Table 1.1 from [1] it follows that the majority of data placed in Table 1.1 were extracted from the NIST/CODATA database (see "CODATA Recommended Values of the Fundamental Physical constants: 1998" [2] and also NIST site).
But from the overview of the last NIST/CODATA adjustment [2] one reads (see page 357)

> . . . On the other hand, in a few cases in the current adjustment a constant that enters the analysis of input data is taken as a fixed quantity rather than an adjusted quantity. An example of the most extreme case is the Fermi coupling constant, which is taken to have the fixed value given by the Particle Data Group (Caso et al., 1998), because the data that enter the current adjustment have a negligible effect on its value. . . .

Indeed in the Review of Particle Physics as of 1998 [3] we have the first of the above two estimates of the Fermi coupling constant. It was obtained with the formula of Marciano-Sirlin [4]. But in 1999 the new formula for the muon lifetime which incorporates the complete 2-loop QED contributions was published [5]. The new estimate for G_F, obtained with Marciano-Sirlin-Ritbergen-Stuart formula were presented repeatedly in the Erler-Langacker mini-reviews of the RPP-2000 [6] and RPP-2002 [1] editions, but not in the corresponding tables (Table 1.1) where the old "CODATA" value is presented repeatedly.
It should be noted also that the Marciano-Sirlin 1988 estimate of the G_F reads

$$G_F/(\hbar c)^3 = 1.16637(2) \times 10^{-5}\,\mathrm{GeV}^{-2}. \tag{3}$$

2. To clarify the situation (i.e. the absence of reprinted misprints in publications) we have performed recalculations of the estimates based on Marciano-Sirlin and Marciano-Sirlin-Ritbergen-Stuart formulas, but with the values of fundamental constants and their correlated uncertainties as adjusted by NIST in 1998 and recommended by CODATA as the unique international source of the numerical data on physical constants. We reproduce exactly the estimates (1) and (2). Thus, the confusion with two different values of the Fermi constant in two different places of the same RPP edition is probably caused by the lack of synchronization between adjustment cycles in PDG collaboration and NIST team. It will be good if NIST could follow the PDG collaboration in physics branches tightly related with particle and astro-particle physics and refresh their adjusted database simultaneously with PDG yearly refreshments.
From the other hand, it is stated in the caption of the Table 1.1 of the RPP-2002 [1]

> . . . The last group of constants (beginning with the Fermi coupling constant) comes from the Particle Data Group. . . ,

and hence the value of the G_F should be the value from Eq.(2) to be consistent with the value quoted in the Erler-Langacker mini-review in RPP-2000 [6] and RPP-2002 [1].

3. In the NIST/CODATA web version of the fundamental constants database correlations the uncertainty of the Fermi coupling constant with uncertainties of the other constants are equal zero or empty. As the data on muon lifetime were not included in the NIST adjustment procedure, the estimate for the Fermi coupling constant should be treated as derived quantity from the values of the recommended fundamental constants and muon lifetime via the Marciano-Sirlin-Ritbergen-Stuart formula. In this case the uncertainty of the Fermi coupling constant will correlate with all constants entering into the formula as well as with the uncertainty of the muon lifetime. In the Table 1 we compare our estimates for correlation coefficients for G_F and m_e, m_μ, m_W, α, $\hbar$, τ_μ, (independently rounded to three decimal digits as in the NIST/CODATA database) with corresponding NIST estimates:

Table 1: Comparison of correlation coefficients

$cor(u(i), u(j))$ for:	Our values	NIST values
$u(G_F)$, $u(m_e)$	-0.008	0.000
$u(G_F)$, $u(m_\mu)$	-0.012	0.000
$u(G_F)$, $u(m_W)$	0.000	none
$u(G_F)$, $u(\alpha)$	0.000	0.000
$u(G_F)$, $u(\hbar)$	-0.009	0.000
$u(G_F)$, $u(\tau_\mu)$	-1.000	none

The “none” in the Tab.1 stands to mark the absence of the W-boson mass – m_W and muon lifetime – τ_μ in the NIST/CODATA database.
It turns out that if we form the sub-matrix of correlation coefficients for the sample

$$G_F,\ m_e,\ m_\mu,\ m_W,\ \alpha,\ \hbar,\ \tau_\mu \tag{4}$$

we will get the matrix that is non-degenerate but not positive definite. It has negative eigenvalue that emerged due to unjustified rounding of the newly calculated correlation coefficients to three decimal digits[1].

Indeed, if we save four decimal digits in our calculations we will obtain:

Table 2: Comparison of correlation coefficients

$cor(u(i), u_(j))$ for:	Our values	NIST values
$u(G_F)$, $u(m_e)$	-0.0085	0.000
$u(G_F)$, $u(m_\mu)$	-0.0118	0.000
$u(G_F)$, $u(m_W)$	0.0001	none
$u(G_F)$, $u(\alpha)$	0.0005	0.000
$u(G_F)$, $u(\hbar)$	-0.0086	0.000
$u(G_F)$, $u(\tau_\mu)$	-0.9999	none

The correlation matrix for the sample (4) formed with our values from Tab.2 and the other values from the NIST/CODATA site became the positive definite as it should be for the non-degenerate correlation matrix by definition. We have no idea where the zero values for correlation coefficients for the Fermi coupling constants came from in the CODATA recommended sample of constants and correlations. We failed to find any related information there, and there is no discussion of this subject in the review [2].

All calculations for this note were produced in the special package ’StandardPhysicalConstants’ [8],[9] in the *Mathematica* system [10].

4. If we trace the evolution of the physical constants presentation in PDG reviews we will see that activities of NIST and PDG in the field of constants adjustment become more and more intertwined. Fundamental constants appeared in the PDG review for the first time in 1967 (see [7], page 7). Starting from the 1984 edition of the Review of Particle Properties (now the Review of Particle Physics) five constants

$$G_F,\ \sin^2\hat{\Theta}(M_z)_{\overline{MS}},\ m_W,\ m_Z,\ \alpha_S(M_z)$$

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[1] Such rounding was motivated by the form of correlations presentation in the NIST/CODATA released resources.

are declared as PDG area of responsibility, and three constants that controlled by PDG are included in the recommended by CODATA sample: namely G_F, weak mixing angle, mass of τ-lepton in MeV, and additional ten τ-related values are based on the PDG value of the τ lepton mass in MeV, (see [2], pages 449, 450).

The continuously increasing experimental precision in fundamental particle physics as well as progress in theoretical calculations, expressing physical observables in terms of the standard model input constants, posed a challenging task for the PDG collaboration, namely: to monitor the crucial parameters of the standard model as thorough as NIST team tried to do with the other fundamental physical constants and give not only the adjusted values of the SM constants with uncertainties but with full covariances (correlations) between them.

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